ST. HENRY'S COLLEGE KITOVU

A'LEVEL PURE MATHEMATICS P425/1 SEMINAR QUESTIONS 2019

ALGEBRA

1. Solve the simultaneous equations:
   (a) $x^2 + y^2 = 5$, $\frac{1}{x^2} + \frac{1}{y^2} = \frac{5}{4}$
   (b) $\frac{x}{y} + \frac{y}{x} = \frac{17}{4}$, $x^2 - 4xy + y^2 = 1$

2. Find the range of values of x for which
   (a) $\frac{2x+1}{x+2} > \frac{1}{2}$.
   (b) $|2x+1| > 7$

3. Resolve into partial fractions
   (a) $\frac{x^3 + x^2 + 4x}{x^2 + x - 2}$
   (b) $\frac{3x^2 + 8x + 13}{(x-1)(x^2 + 2x + 5)}$
   (c) $\frac{2x^3 + 2x^2 + 2}{(x+1)^2(x^2+1)}$

4. Solve the following equations:
   (a) $2^{3x+1} = 5^{x+1}$
   (b) $9^x - 4(3^x) + 3 = 0$
   (c) $\log_x 9 + \log_{x^2} 3 = 2.5$
   (d) $\sqrt{2x-1} - \sqrt{x-1} = 1$
   (e) $\begin{aligned} 2x + 3y + 4z &= 8 \\ 3x - 2y - 3z &= -2 \\ 5x + 4y + 2z &= 3 \end{aligned}$

5. Find:
   (a) The three numbers in arithmetic progression such that their sum is 27 and their product is 504
   (b) The three numbers in a geometrical progression such that their sum 39 and their product is 729.
   (c) The sum of the last three terms of a geometrical progression having n terms is 1024 times the sum of the first three terms of the progression. If the third term is 5, find the last term.

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(d) Prove by induction that $1^3+2^3+....+n^3=\frac{1}{4}n^2(n+1)^2$ and deduce that

$1^3+3^3+5^3....+(2n+1)^3=(n+1)^2(2n^2+4n+1)$

6. Expand:

(a) $\frac{7+x}{(1+x)(1+x^2)}$ in ascending powers of x as far as the term in x^4.

(b) $\left(1-\frac{3}{2}x-x^2\right)^5$ in ascending powers of x as far as the term in x^4.

(c) Find the term independent of x in the expansion of $\left(2x+{}^1\!/_{x^2}\right)^{12}$ in descending powers of x and find the greatest term in the expansion when $x=\frac{2}{3}$.

(d) Find by binomial theorem, the coefficient of x^8 in the expansion $(3-5x^2)^{1/2}$ in ascending powers of x.

(e) In the binomial expansion of $(1+x)^{n+1}$, n being an integer greater than two, the coefficient of x^4 is six times the coefficient of x^2 in the expansion $(1+x)^{n-1}$. Determine the value of n.

7. (a) Without using the calculator, simplify $\frac{\left(\cos\left(\frac{\pi}{9}\right)+i\sin\left(\frac{\pi}{9}\right)\right)^4}{\left(\cos\left(\frac{\pi}{9}\right)-i\sin\left(\frac{\pi}{9}\right)\right)^5}$

(b) In a quadratic equation $z^2+(p+iq)z+3i=0$. p and q are real constants. Given that the sum of the squares of the roots is 8. Find all possible pairs of values of p and q.

8. (a) How many different arrangements of letters can be made by using all the letters in the word contact? In how many of these arrangements are the vowels separated?
(b) In how many ways can a team of eleven be picked from fifteen possible players.

9. (a) If α and β are the roots of the equation $x^2-px+q=0$, form the equation whose roots are $\frac{\alpha}{\beta^2}$ and $\frac{\beta}{\alpha^2}$.

(b) If α and β are the roots of the equation $x^2+bx+c=0$, form the equation whose roots are $\frac{1}{\beta^3}$ and $\frac{1}{\alpha^3}$. If in the equation above $\alpha\beta^2=1$, prove that $a^3+c^3+abc=0$

10. (a) If $z=x+iy$ and $\bar{z}$ is the conjugate of z, find the values of x and y such that $\frac{1}{z}+\frac{2}{\bar{z}}=1+i$

(b) If x, y, a and b are real numbers and if $x+iy=\frac{a}{b+\cos\theta+i\sin\theta}$. Show that $(b^2-1)(x^2+y^2)+a^2=2abx$.

(c) If n is an integer and $z=cos\theta+sin\theta$, show that $2\cos n\theta=z^n+\frac{1}{z^n}$, $2i\sin n\theta=z^n-\frac{1}{z^n}$.

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Use the result to establish the formula $8cos^4\theta = cos4\theta + 4cos2\theta + 3$.

(e) If z is a complex number and $\left|\frac{z-1}{z+1}\right| = 2$, find the equation of the curve in the Argand diagram on which the point representing z lies.

TRIGONOMETRY

11. If $sin\theta + sin\beta = a$ and $cos\theta + cos\beta = b$, show that $\cos^2\left(\frac{\theta-\beta}{2}\right) = \frac{1}{4}\left(a^2+b^2\right)$

12. Show that $\sin 7x + \sin x - 2\sin 2x \cos 3x = 4\cos^3 3x$

13. If A, B and C are angles of a triangle, show that:

(i) $\cos A + \cos(B-C) = 2\sin B \sin C$

(ii) $\cos\frac{C}{2} + \sin\frac{A-B}{2} = 2\sin\frac{A}{2}\cos\frac{B}{2}$

14. Express $y = 8\cos x + 6\sin x$ in form of $R\cos(x-\alpha)$ where R is positive and α is acute. Hence find the maximum and minimum values of $\frac{1}{8\cos x + 6\sin x + 15}$ and the corresponding angle respectively.

15. Show that:

(a) $\tan^{-1} x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$

(b) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

(c) Find x if $\tan^{-1} x + \tan^{-1}(1-x) = \tan^{-1}\left(\frac{4}{3}\right)$

16. (a) Show that $\cos 4\theta = \frac{\tan^4\theta - 6\tan^2\theta + 1}{\tan^4\theta + 2\tan^2\theta + 1}$

(b) Solve the equation $8\cos^4 x - 10\cos^2 x + 2 = 0$ for x in the range of $0° \leq x \leq 180°$

17. (a) If $\tan\theta = \frac{1}{p}$ and $\tan\beta = \frac{1}{q}$ and $pq = 2$p, show that $\tan(\theta+\beta) = p+q$

(b) Show that $\sin 2A + \cos 2A = \frac{(1+\tan A)^2 - 2\tan^2 A}{1+\tan^2 A}$

18. If α, β and γ are all greater than $\frac{\pi}{2}$ and less than 2π and $\sin\alpha = \frac{1}{2}$, $\tan\beta = \sqrt{3}$, $\cos\gamma = \frac{1}{\sqrt{2}}$.

Find the value of $\tan(\alpha+\beta+\gamma)$ in surd form.

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19. Solve for x in the range $0°$ to $360°$

(a) $3\cos^2 x - 3\sin x\cos x + 2\sin^2 x = 1$

(b) $4\cos x = 3\tan x + 3\sec x$

20. Prove that $4\cos\theta\cos 3\theta + 1 = \frac{\sin 5\theta}{\sin\theta}$. Hence find all the values of θ in the range $0°$ to $180°$ for which $\cos\theta\cos 3\theta = \frac{-1}{2}$

VECTORS

21. The coordinates of the points A and B are (0,2,5) and (-1,3,1) and the equation of the line L is $\frac{x-3}{2} = \frac{y-2}{-2} = \frac{z-2}{-1}$

(i) Find the equation of the plane containing the point A and perpendicular to L and verify that B lies in the plane.

(ii) Show that the point C in which L meets the plane is (1,4,3) and find the angle between CA and CB

22. (a) A body moves such that its position is given by $OP = (3sint)i + (3cost)j$ where O is the origin and t is the time. Prove that the velocity of the particle when at P is perpendicular to OP.

(b) The lines L_1 and L_2 have Cartesian equations $\frac{x}{1} = \frac{y+2}{2} = \frac{z-5}{-1}$ and $\frac{x-1}{-1} = \frac{y+3}{-3} = \frac{z-6}{1}$. Show that L_1 and L_2 intersect and find the coordinates of the point of intersection.

23. (a) Find the acute angle between the lines whose equations are $\frac{x-2}{-4} = \frac{y-3}{3} = \frac{z+1}{-1}$ and $\frac{x-3}{2} = \frac{y-1}{6} = \frac{z+1}{-5}$.

(b) The points A and B have coordinates (1,2,3) and (4,6,-2) respectively and the plane has equation $x + y - z = 24$. Determine the equation of the line AB, hence the angle this line makes with the plane.

24. (a) Find the perpendicular distance of the line $\frac{x-5}{1} = \frac{y-6}{2} = \frac{z-3}{4}$ from the point (-6,-4,-5).

(b) Find the shortest distance between the two skew lines $\frac{x+1}{1} = \frac{y-2}{2} = \frac{z-3}{1}$ and $\frac{x}{2} = \frac{y+1}{1} = \frac{z-1}{3}$ respectively.

(c) Find the perpendicular distance of the plane $2x - 14z + 5z = 10$ from the origin.

25. (a) Show that the line $\frac{x-2}{2} = \frac{y-2}{-1} = \frac{z-3}{3}$ is parallel to the plane$4x - y - 3z = 4$ and find the perpendicular distance from the line to the plane.

(b) Find the Cartesian equation of the line of intersection of the two planes $2x - 3y - z = 1$ and $3x + 4y + 2z = 3$.

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26. (a) Find the Cartesian equation of the plane containing the point (1,3,1) and parallel to the vectors $\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$

(b) Find the Cartesian equation of the plane containing the points (1,2,-1), (2,1,2) and (3,-3,3).

27. Given the points A, B and C with coordinates (2,5,-1), (3,-4,2) and C(-1,2,1). Show that ABC is a triangle and find the area of the triangle ABC

28. (a) Find the angle between the parallel planes $3x + 2y - z = -4$ and $6x + 4y - 2z = 6$.

(b) Find the acute angle between the planes $2x + y + 3z = 5$ and $2x + 3y + z = 7$

29. The points A and B have coordinates (2,1,1) and (0,5,3) respectively. Find the equation of the line AB. If C is the point (5,-4,2). Find the coordinates of D on AB such that CD is perpendicular to AB. Find the equation of the plane containing AB and perpendicular to the line CD.

30. (a) Given that $OP = \begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix}$ and $OQ = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$, find the coordinates of the point R such that $\overline{PR} = \overline{PQ} = 1:2$ and the points P, Q and R are collinear.

(b) A and B are the points (3,1,1) and (5,2,3) respectively, and C is a point on the line $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$. If angle BAC=$90^0$, find the coordinates of C

ANALYSIS

31. Differentiate from first principles

(a) $y = \tan^{-1} x$

(b) $y = ax^n$

(c) $y = \sin 3x$

32. Find the derivative of:

(a) $y = 5\sin^{-1}(4x)$

(b) $y = \tan^{-1}\left(\dfrac{1+\tan x}{1-\tan x}\right)$

(c) $y = \dfrac{\sin x}{x^2 + \cos x}$

(d) $y = \sqrt{\dfrac{x}{1+x}}$

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33. Find:

(a) $\int \sin^{-1} x$

(b) $\int \frac{dx}{x^2+4x+13}$

(c) $\int \frac{dx}{x\log_e x}$

(d) $\int \frac{dx}{(1+x^2)\tan^{-1} x}$

(e) Show that $\int_0^2 \sqrt{\frac{x}{4-x}}dx = \pi - 2$

(f) Show that $\int_1^{10} x\log_{10} x = 50 - \frac{99}{4\ln 10}$

34. (a) If $x = t^3$ and $y = 2t^2$. Find $\frac{dy}{dx}$ in terms of t and show that when $\frac{dy}{dx} = 1$, $x = 2$ or $x = \frac{10}{27}$

(b) If $y = \frac{2t}{1+t^2}$ and $x = \frac{1-t^2}{1+t^2}$, find $\frac{d^2y}{dx^2}$ in terms of t

35. Given that:

(a) $y = \sqrt{4+3\sin x}$, show that $2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + y^2 = 4$

(b) $y = e^{2x}\cos 3x$, show that $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$

(c) $y = \left(x + \sqrt{1+x^2}\right)^p$, show that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - p^2y = 0$

(d) $y = \sin(\log_e x)$, show that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

36. (a) Find the volume generated when the area enclosed by the curve $y = 4x - x^2$ and the line $y = 2x$ is rotated completely about the x – axis.

(b) Find the area contained between the two parabolas $4y = x^2$ and $4x = y^2$.

(c) Find the area between the curve $y = x^3$, the x – axis and the lines $y = 1, y = 8$.

(d) Find the area of the curve $x^2 + 3xy + 3y^2 = 1$

(e) Show that in the solid generated by the revolution of the rectangular hyperbola $x^2 - y^2 = a^2$ about the x – axis, the volume of the segment of height a from the vertex is $\frac{4}{3}\pi a^3$

37. (a) A right circular cone of semi – vertical angle θ is circumscribed about a sphere of radius R. show that the volume of the cone is $V = \frac{1}{3}\pi R^3(1+\operatorname{cosec}\theta)^3\tan^2\theta$ and find the value of θ when the volume is minimum.

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(b) Water is poured into a vessel, in the shape of a right circular cone of vertical angle 90°, with the axis vertical, at the rate of $125\text{cm}^3/s$. At what rate is the water surface rising when the depth of the water is 10cm?

38. Sketch the curve $y = \frac{x}{x+2}$. Find the area enclosed by the curve, the lines $x = 0, x = 1$ and the line $y = 1$. Also find the volume generated when this area revolves through 2π radians about the line $y = 1$.

39. Solve the differential equations below:

(a) $\frac{1}{3x}\frac{dy}{dx} + \cos^2 y = 1$, when $x = 2$ and $y = \frac{\pi}{4}$

(b) $(x-y)\frac{dy}{dx} = x + y$, when $x = 4$ and $y = \pi$

(c) $\frac{dy}{dx} + 3y = e^{2x}$, when $x = 0$ and $y = \frac{6}{5}$

40. In a certain type of chemical reaction a substance A is continuously transformed into a substance B. throughout the reaction, the sum of the masses of A and B remains constant and equal to m. The mass of B present at time t after the commencement of the reaction is denoted by x. At any instant, the rate of increase of mass of B is k times the mass of A where k is a positive constant.

(a) Write down a differential equation relating x and t

(b) Solve this differential equation given that $x = 0$ and $t = 0$. Given also that $x = \frac{1}{2}m$ when $t = \ln 2$, determine the value of k and show that at time t, $x = m(1 - e^{-t})$. Hence find:

(i) The value of x (in terms of m) when $t = 3\ln 2$

(ii) The value of t when $x = \frac{3}{4}m$

GEOMETRY

41. (a) Find the equation of a line which makes an angle of 150° with the x – axis and y – intercept of -3 units.

(b) Find the acute angle between the lines $3y - x = 4$ and $6y - 3x - 5 = 0$

(c) OA and OB are equal sides of an isosceles triangle lying in the first quadrant. OA and OB make angles θ_1 and θ_2 with x – axis respectively. Show that the gradient of the bisector of the acute angle AOB is $\operatorname{cosec}\theta - \cot\theta$ where $\theta = \theta_1 + \theta_2$

(d) Find the length of the perpendicular from the point P(2,-4) to the line $3x + 2y - 5 = 0$

42. (a) Find the equation of the circle with centre (4,-7) which touches the line $3x + 4y - 9 = 0$

(b) Find the equation of the circle through the points (6,1), (3,2), (2,3)

(c) Find the equation of the circumcircle of the triangle formed by three lines $2y - 9x + 26 = 0$, $9y + 2x + 32 = 0$ and $11y - 7x - 27 = 0$

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43. (a) Find the length of the tangent from the point (5,6) to the circle $x^2+y^2+2x+4y-21=0$.

(b) Find the equations of the tangents to the circle $x^2+y^2=289$ which are parallel to the line $8x-15y=0$

(c) Find the equation of the circle of radius $12\frac{4}{5}$ which touches both the lines $4x-3y=0$ and $3x+4y-13=0$ and intersects the positive y – axis.

(d) A circle touches both the x – axis and the line $4x-3y+4=0$. Its centre is in the first quadrant and lies on the line $x-y-1=0$. Prove that its equation is $x^2+y^2-6x-4y+9=0$

44. Find the equations of the parabolas with the following foci and directrices:

(i) Focus (2,1), directrix $x=-3$

(ii) Focus (0,0), directrix $x+y=4$

(iii) Focus (-2,-3), directrix $3x+4y-3=0$

45. (a) Show that the curve $x=5-6y+y^2$ represents a parabola. Find its focus and directrix, hence sketch it.

(b) Find the equation of the normal to the curve $y^2=4bx$ at the point $P(bp^2,2bp)$. Given that the normal meets the curve again at $Q(bq^2,2bq)$, prove that $p^2+pq+2=0$

46. (a) Show that the equation of the normal with gradient m to the parabola $y^2=4ax$ is given by $y=mx-2am-am^3$.

(b) P and Q are two points on the parabola $y^2=4ax$ whose coordinates are $P(ap^2,2ap)$ and $Q(aq^2,2aq)$ respectively. If OP is perpendicular to OQ, show that $pq=-4$ and that the tangents to the curve at P and Q meet on the line $x+4a=0$

47. (a) A conic is given by $x=4\cos\theta$, $y=3\sin\theta$. Show that the conic is an ellipse and determine its eccentricity

(b) Given that the line $y=mx+c$ is a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, show that $c^2=a^2m^2+b^2$. Hence determine the equations of the tangents at the point (-3,3) to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$.

48. (a) Show that the locus of the point of intersection of the tangents to an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which are at right angles to one another is a circle $x^2+y^2=a^2+b^2$.

(b) The normal to the ellipse $x^2+y^2=100$ at the points A(6,4) and B(8,3) meet at N. If P is the mid – point of AB and O is the origin, show that OP is perpendicular to ON.

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49. (a) P is a point $(ap^2, 2ap)$ and Q the point $(aq^2, 2aq)$ on the parabola $y^2 = 4ax$. The tangents at P and Q intersect at R. Show that the area of triangle PQR is $\frac{1}{2}a^2(p-q)^3$

(b) The normal to the parabola $y^2 = 4ax$ at $P(ap^2, 2ap)$ meets the axis of the parabola at M and MP is produced beyond P to Q so that MP = PQ. Show that the locus of Q is $y^2 = 16a(x+2a)$

50. (a) The normal to the rectangular hyperbola $xy = 8$ at the point (4,2) meets the asymptotes at M and N. Find the length of MN

(b) The tangent at P to the rectangular hyperbola $xy = c^2$ meets the lines $x - y = 0$ and $x + y = 0$ at A and B and Δ denotes the area of triangle OAB where O is the origin. The normal at P meets the x – axis at C and the y – axis at D. if Δ_1 denotes the area of the triangle ODC. Show that $\Delta^2\Delta_1 = 8c^6$

END

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KIIRA COLLEGE BUTIKI

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

LOCK DOWN REVISION QUESTIONS

SECTION A (40 marks)

1. Solve the inequality

   $\frac{x(x+2)}{x-3} \leq x + 1$ (5 marks)

2. Show that the line $\frac{x-2}{2} = \frac{y-2}{-1} = \frac{z-3}{3}$

   Is parallel to the plane 4x-y-3z=4 and find the perpendicular distance of the line from the plane. (5 marks)

3. Solve the equation

   2tan x-3cot x=1

   For $0^o \leq x \leq 360^o$ (5 marks)

4. Calculate the co-ordinates of the point of the intersection of the curve

   $\frac{x}{y} + \frac{6y}{x} = 5$ and $2y = x - 2$ (5 marks)

5. The tangent to the curve $y = 2x^2 + ax + b$ at the point (-2,11) is perpendicular to the line $2y = x + 7$. Find the value of a and b. (5 marks)

6. Evaluate $\int_{o}^{\frac{\pi}{3}} cos3xcos2xdx$ (5 marks)

7. Given that φ in a root of the equation $x^2 - 2x + 3 = 0$ show that $\varphi^3 = x - 6$

   (5 marks)

8. A spherical balloon is being inflated by gas being pumped at the constant rate of 200cm^3 per second. What is the rate of increase of the surface area of the ballon when its radius is 100cm? (5 marks)

SECTION B (60 MARKS)

9. (a) $If\ (x+1)^2\ is\ factor\ of\ 2x^4+7x^3+6x^2+Ax+b,$ find the value of A and B. (5 marks)

(b) Prove that, if the equations $x^2+ax+b=0\ and\ cx^2+2ax-3b=0$ have a common root and neither a and b is zero, then

$b=\frac{5a^2(c-2)}{(c+3)^2}$ (7 marks)

10. (a) Given that $y=loge(\frac{3+4cosx}{4+cosx})$ find $\frac{dy}{dx}$ in the simplest form. (7 marks)

(b) $If\ y=e^{4x}cos3x, prove\ that\ \frac{d2y}{dx^2}-8\frac{dy}{dx}+25y=o$ (7 marks)

11. (a) Given that $z=cos\theta_{sin\theta}, where\ \theta\neq\pi, show\ that\ \frac{2}{1+z}=1-itan\frac{1}{2}\theta.$

(6 marks)

(b) The polynomial $p(z)=z^4-3z^3+7z^2+21z-26\ has\ 2+3i$

$as\ one\ of\ the\ roots. Find\ the\ other\ three\ roots\ of\ the\ equation\ p(z)=o$

(6 marks)

12. (a) A right circular cone with semi vertical angle θ is inscribed in a sphere of radius γ, with its vertex and rim of its base on the surface of the sphere. Prove that its volume is $\frac{8}{3}\pi r^3cos^4\theta sin^2\theta.$ (6 marks)

(b) If r in constant and θvaries, show that the limits within which this volume must lie is $0<v<\frac{32\pi r^3}{81}1$ (6 marks)

13. (a) In any triangle ABC, prove that $tan\frac{1}{2}(B-C)=\left(\frac{b-c}{b+c}\right)tan\frac{1}{2}(B+C)$

(6 marks)

(b) In a particular triangle the angle A is 51^o and b=3c. Find the angle B to the nearest degree. The area of this triangle in $0.47m^2$. Find side a to three decimal places.

14. (a) The points A and B have position vector i-2jtk and 2ijk respectively. Given that $0c=\boldsymbol{\lambda}$OA $+\mu OB\ and\ OC\ is\ perpendicular\ to\ OA, find\ the$ Ratio of $\boldsymbol{\lambda}$ to μ.

Write down the vector equation of the line, L through A which is perpendicular to OA. Find the position vector of P, the point of intersection of Land OB. (12 marks)

15. (a) Determine the equation of the normal to the eclipse $x\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at a point p(a cos θ, bsin θ. (6 marks)

(b) If the normal at p meets the x – axis at A and the y – axis at B, Find the locus of the midpoint of AB. (6 marks)

16. (a) Solve the differential equation $x\frac{dy}{dx}=y+x^2(cosx+sinx),$ given that $y=o\ when\ x\frac{\pi}{2}$ (5 marks)

(b) The rate of decay of a radioactive substance is proportional to the amount A remaining at any time t. If initially the amount was Ao and if the time taken for the amount of substance to become ½ Ao is T, find A at that time.

Find the time taken for the amount remaining to be reduced to $\frac{1}{20}Ao$

(7 marks)

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PURE MATHEMATICS

Paper 1

LOCK DOWN REVISION QUESTIONS 2020

SECTION A (40 MARKS)

1. Solve the simultaneous equations;

   $x + y = 4$

   $x^2 + y^2 - 3xy = 76$ *(05 marks)*

2. Solve the equation; $\sqrt{3}\ \sin\theta - \cos\theta + 2 = 0$ for $0 < 0 < 2\pi$. *(05 marks)*

3. Find the equations of the lines which pass through the point A(3, -2) and makes an angle θ with the line $2x - 3y - 4 = 0$, where $\tan\theta = 2$. *(06 marks)*

4. Show that $\dfrac{(\sqrt{3} - i)^5}{\sqrt{3} + i} = -16$ *(05 marks)*

5. If $y = A\,x^k$, where A and K are non – zero constants , find the values of K such that; $x^2 \dfrac{d^2y}{dx^2} + 2x\dfrac{dy}{dx} - 2y = 0$ *(05 marks)*

6. Using the substitution $x = e^t$, evaluate the $\int_1^e \dfrac{3 - 1nx}{x^2}\ dx.$ *(05 marks)*

7. Given that A and B are points whose position vectors are **a** = 2**i** + **k** and **b** = **i** - **j** + 3**k** respectively.
   Determine the position vector of the point that divides AB in the ratio -4 : 1 *(04 marks)*

8. Find the area bounded by the three curves $y = x^2$, $y = \frac{1}{4}x^2$ and $y = \frac{1}{x^2}$ in the first quadrant. *(05 marks)*

SECTION B (60 MARKS)

9. (a) Find $\int \frac{1}{x^3\ \sqrt{x^2-4}}\,dx$ *(06 marks)*

(b) Evaluate $\int_3^4 \frac{x^3}{x^2-x-2}\,dx$ *(06 marks)*

10. (a) The eighth term of an arithmetic progression is twice the fourth term, and the sum of the eight terms is 30. Find the

(i) first four terms, *(06 marks)*

(ii) sum of the first 12 terms, of the progression *(02 marks)*

(b) Find the number of ways in which the letters of the word STATISTICS can be arranged in a straight line so that,

(i) the last two letters are both Ts. *(02 marks)*

(ii) all the three **Ss** must be together *(02 marks)*

11. (i) Given that the roots of the equation $ax^2 + bx + c = 0$ are α and β. Show that $a^2 = b^2 - 4ac$ if $\alpha - \beta = 1$. *(06 marks)*

(ii) Find a quadratic equation whose roots are $(\alpha + \alpha\beta)$ and $(\beta + \beta\alpha)$ in terms of a, b and c. *(06 marks)*

12. (a) Differentiate with respect to x,

(i) $2^{\cos x^2}$ *(03 marks)*

(ii) $\log_e \left(\frac{(1+x)\,e^{-2x}}{1-x}\right)^{½}$ *(03 marks)*

(b) (i) Determine the equation of the normal to the curve $y=\frac{1}{x}$ at the point x = 2. *(03 marks)*

(ii) Find the coordinates if the other point where the normal meets the curve again

(03 marks)

13. (a) Given the points A (3, 1, 2) and B (2, -2, 4), find the sin e of the angle BOC.
Hence determine the area of triangle AOB. Where O is the origin. *(06 marks)*

(b) Show that the line $\frac{x-2}{2} = \frac{2-y}{1} = \frac{3-z}{-3}$ is parallel to the plane r .(4**i** – **j** – 3**k**) = 4.
Hence find the perpendicular distance between the line and the plane. *(06 marks)*

14. (a) Show that for any triangle ABC,
$\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ *(05 marks)*

(b) Prove that $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$, hence solve the equation

$\tan(x - 45^0) = 6\tan x$, where $-180^0 \leq x \leq 180^0$ *(07 marks)*

15. (a) Find the equation and radius of a circle passing through the points A (0,1), B (0, 4) and C (2 , 5). *(05 marks)*

(b) A circle passes through the point P(1, -4) and is tangent to the y-axis. If its radius is 5 units, find its equation *(07 marks)*

16. (a) Given that y = 0 when x = 0, solve the equation $\frac{dy}{dx} = 2y + 3$, expressing y as a function of x. *(05 marks)*

(b) When a uniform rod is heated it expands in such a way that the rate of increase of its length, l, with respect to the temperature, θ^0 C, is proportional to the length. When the temperature is 0^0C the length of the rod is L. Given that the length of the rod has increased by 1% when the temperature is 20^0C, find the value of θ at which the length of the rod has increased by 5%. *(07 marks)*

WAKISSHA JOINT MOCK EXAMINATIONS 2015
UGANDA ADVANCED CERTIFICATE OF EDUCATION
MARKING GUIDE
P425/1
MATHEMATICS
PAPER 1
JULY/AUGUST 2015

1. $X=5+\frac{\sqrt{3}}{2}\cos\Theta$ ………… (1)		
$Y=-3\frac{\sqrt{3}}{2}\sin\Theta$ ………….. (2)		
From 1 $X-5=\frac{\sqrt{3}}{2}\cos\Theta$		
$\left(\frac{2X-10}{\sqrt{3}}\right)=\cos\Theta$		
$\sqrt{3}$		
$\cos^2\Theta=\left(\frac{2X-10}{\sqrt{3}}\right)^2$		
From (2) $Y+3=\frac{\sqrt{3}}{2}\sin\Theta$		
$\sin\Theta=\frac{2(Y+3)}{\sqrt{3}}$	B1	B1 – for $Co^2\Theta$ and $\sin\Theta$
$\sin^2\Theta=\frac{(2Y-6)^2}{3}$		Or
	M1	$(x-5)^2=\frac{3}{4}\cos^2\Theta$………(1) B1
$\cos^2\Theta+\sin^2\Theta=2\frac{(2X-10)^2}{3}+\frac{(2Y-6)^2}{3}$		$(y+3)2=\frac{3}{4}\sin^2\Theta$………(2)
$(2X-10)^2+(2Y-6)^2=3$		(1) + (2)
$4X^2-40X+100+4Y2-24Y+36=3$		$(x-5)^3+(y+3)^2=\frac{3}{4}$ M1
$4X^2+4Y^2-40X-24Y+133=0$		$4x2+4y2-40x-24y+130=0$ A1
$X^2+Y^2-10X+6Y-133$ -0		Is locus of a circle B1
$(X-5)^2+(Y+3)^2=\frac{3}{4}$		The radius = $\frac{\sqrt{3}}{2}$ and centre (5, -3) B1
Locus is a circle with centre	B1	
(5 – 3) and radius $\sqrt{3}$	B1	
	05	

2. 3^{2n+2}-8n-9.		
Let au= 3^{2n+2}-8n-9.		
For n=1 $a_1=3^4$-8-9=64	B1	
thus divisible.		
For n=k.		
ak=3^{2X+2}-8k-9.$\Rightarrow$9k = $3^{2k}.3^2$-8k-9		
9k=9($3^{2\pi}$) - 8k-9.	B1	
32k = $\frac{9k+8k+9}{9}$		
For = k+1		
9R+1 = $3^{2(k+1)+2}$-8(k+1)-9		
9R+1=$3^{2\pi+4}$-8k –8-9$\Rightarrow$9K+1=81(3^{4k}) -8k - 17		
Subst.(3) in (4)	B1	
81($\frac{9k+8k+9}{9}$).8k-17		
9k+1=9ak+72R+81-8K-17.		
9K+1= 9AK+64R+64.		
9K+1 = 64($\frac{9ak}{64}$+k+1)		
Ak+1=64Bfor B=$\frac{9ak}{64}$+C+1		
Since its divisible for n= π41,n=c thus this also divisible for all integral values of n.	B1	
	05	
3. 6Cos2X+777=7Sin2X using t = tan x		
$6\left(\frac{1-t^2}{1+t^2}\right)+7=7\left(\frac{2t}{1+t^2}\right)$	M1	For substitution
6(1-t^2) +7(1+t^2) =14t.$\Rightarrow t^2$-14t+13=0		
(t-1) (t-13)=0		
t=1 t=13	M1	For method
for t=1	A1	
tan X=1 X=45^0,225^0		
For t=13	A1	
tan x=13 x=85.6^0, 265$^-$ 6^0		
Or	A1	
7Sin2x-6Cos2X=7		
R Sin(2x∝)= 7Sin2X-6CosR		
R Cos∝=7 and R Sin∝=6		
R=$\sqrt{49}+36$ = S85 tan∝=$\frac{6}{7}$		
$\sqrt{85}$ Sin(2x-∝)=7		
Sin(2x-∝)=$\frac{7}{\sqrt{85}}$		
2x = $\sin^{-1}\left(\frac{7}{85}\right)$+$\tan^{-1}\left(\frac{6}{7}\right)$X =		
A1A1 = 05		

4. $\|X+3a\|<2\|X-2a\|$ $(X+3a)^2<4(x-2a)^2$ $X^2+6ax+9a2<4(X^2-4aX+4a)$ $3x^2-22ax+7a^2>0$ $(3x-a)(x-7a)>0$	A1	

	$X<\frac{1}{3}a$	$\frac{1}{3}a<x<7a$	$X>7a$
3x-a	-	+	+
x-7a	-	-	+
	+	-	+

$\therefore$ The solution is $X<\frac{1}{3}a$ and $x>7a$	A1	A1 for each range of solution
5. $\int_{\sqrt{3}}^{\infty} \frac{dx}{x\sqrt{1+x^2}}$ Method 1 Let $u=\sqrt{1+x^2}$		

x	$\sqrt{3}$	∞
u	2	∞

$du = \frac{1}{2}(1+x2)^{-1/2}.2xdx.$	B1	
$du = \frac{dx}{\sqrt{1+x^2}}dx$ $I = \int_2^{\infty} \frac{1}{U^2-1}\frac{U}{X}\ du$ $I = \int_2^{\infty} \frac{1}{U^2-1}\ du$ Let $\frac{1}{U^2-1} = \frac{A}{u-1} + \frac{B}{u+1}$		
$A=\frac{1}{2}$, $B=\frac{-1}{2}$	B1	
$I = \frac{1}{2}\left[In\left(\frac{u-1}{u+1}\right)\right]_2^{\alpha}$	M1	
$=\frac{1}{2}(0-in\frac{1}{3})$ $=\frac{1}{2}in3$	A1	
	05	
Alternative method. $$\int_{\sqrt{3}}^{\infty} \frac{dx}{x\sqrt{1+x^2}}$$ Let x=tan u		
$dx= Sec^2udu$	B1	(for only changing variablesi.e x to U because it can be worked without involving limits until toward end then we bring limits.
$I=\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{\tan u\ secu}$	M1	
$=\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{Cos\ U}{Sin\ U}\frac{1}{CosU}\ du.$ $=\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} Cosec\ u\ du$		

$\left[in(Tan\frac{u}{2})\right]$ $=(0\text{-in}\frac{1}{\sqrt{3}}$ $=\frac{1}{2}\ln 3$	B1 A1	
6. (i-λi+4k).(2i-4j+6k)=0 4λ =-26 λ =$\frac{-13}{2}$ or -6.5 Using ratio theorem. OP=$(\frac{-3}{-3+2})$a +$(\frac{-3}{-3+2})$b 2(i-2j+4k)-2(3i-4j+6k) =3i-6j+12k-6i+8j-12k = -3i+2j	M1 M1 A1 **05**	
7. Let ar, ar^2, ar^3, be the age of the children in order and that of Pondo respectively. ar, ar^2, ar^3=140 $a(i+r+r^2+r^3)$=140 ………..i and a+ar =14 a(i+r) = 14………………..ii ar^2, ar^3=126 ⇒ $ar^2(1+r)$=126……….iii (i)/(ii)gives r^2=9 r=+-3 r=3 a(1+3)=14 a=$\frac{7}{2}$ is a root (r+1)(r+3)(r-3)=0 ⇒ r=3, r=-1, and r=-3 are roots. r=3 subset for r=3 in equation (ii) 4a=14 a=$\frac{7}{2}$ Pando's age = ar^3=$\frac{7}{2}$x3^3 =27x$\frac{7}{2}$ giving 94.5years.		

8. YCos2X $\frac{dy}{dx}$= tan x+2, at y=0, X=$\frac{\pi}{4}$ y$\frac{dy}{dx}$= $\frac{\text{tan x+2}}{\text{Cos}^2\text{x}}$ y$\frac{dy}{dx}$=(tanx+2)sec^{2}x ydy = $\int \tan x\ sec^2 xdx + 2sec2xdx$ $\frac{y^2}{2}$ =$\frac{1}{2}$tan^{2}x+ tan x+c At y=0, x=$\frac{\pi}{4}$ 0= $\frac{1}{2}$(tan$\frac{\pi}{4}$)^2+2tan$\frac{\pi}{4}$+c = $\frac{1}{2}$(1)^2+2(1)+c C=$\frac{-5}{2}$ $\frac{y^2}{2}$=$\frac{1}{2}$tan^{2}x+ tan x-$\frac{-5}{2}$		
9.		

10. (a)(i) Let Z=x+iy $\|Z-i\|<3$ $\|x+(y-1)i\|<3$ $X^2+(y-1)^2<3^2$ This is a circle with centre (0,1) and radius r=3 (ii)This $\pi \leq \arg(z,2) \leq \pi$ This is a region of half lines from (2,0) between $\frac{1}{3}\pi$	B1B1 B1	B1 for stating centre (0,1) B1 for stating $r \leq 3$ For correct sketch and shading
(b) Z =X+IY $\mathrm{Re}\,\frac{Z+i)}{Z+2}= 0$ $\mathrm{Re}\,\frac{x+(y+1)i}{(x+2)+yi}$ $\mathrm{Re}\,\frac{x+(y+1)i\ (x+2)-yi}{(x+2)+yi\ (x+2)-yi}=0$ $\mathrm{Re}\,\frac{x(x+2)-xyi+i(x+2)(y+1)+y(y+1)}{(x+2)^2+y^2}=0$ $\frac{X(x+2)+y(y+1)}{(x+2)^2+y^2}$ $X^2+2x+y^2+y=0$ $(x+1)^2-1+(y+\frac{1}{2})^2-\frac{1}{4}=0$ $(x+1)^2+(y+\frac{1}{2})^2=\frac{5}{4}$ Z=X+iy … on a circle of centre $(-1,\frac{1}{2})$ and radius $R=\frac{\sqrt{5}}{2}$		

11. Sin30=Sin(2Θ+ Θ) = Sin2ΘCosΘ +Cos2Θ Sin Θ (2SinΘCosΘCosΘ)+(1-2Sin^{2}Θ)sinΘ 2SinΘ(1-Sin2Θ)+(1-2Sin^{2}Θ)SinΘ 2SinΘ-2Sin^{3}Θ+SinΘ-2Sin^{3}Θ Sin^{3}Θ = 3SinΘ-4Sin^{3}Θ Let Sin3Θ = $\frac{p}{q}$ ⇒$\frac{p}{q}$=3SinΘ-4 Sin^{-3}Θ 4q Sin^{-3}Θ-3qSinΘ+P=0 If SinΘ= x ⇒4qx^3-3qx+p=8x^3-6x-1=0 Q=2 and P=-1 Sin3Θ $=-\frac{1}{2}$		
3Θ = Sin^{-1}($-\frac{1}{2}$) 210^0,330^0,570^0,690^0,930^0 Q=70^0,110^0,190^0,230^0,310^0 X=Sin700=0.9397. X= Sin1900=-0.1736 X=Sin2300=-0.7660		
12. (a) Y2=4X-8 Can be written as Y2=4(x-2) =4.1(x-2) =4ax X= x-2 a=1 Y^2=4(x-2) is the image of y^2=4x under translation vector $\binom{2}{0}$ New focus = $\binom{1}{0}\binom{2}{0}\binom{3}{0}$ Focus (3,0) New direction = -1+2 X=1		

(b) At P(ap2,2ap) X=ap2 y=2ap $\frac{dx}{dp}$=2aP $\frac{dy}{dp}$=2a. $\frac{dy}{dx}=\frac{2a}{2ap}\frac{1}{p}$ Gradient of tangent at P $=\frac{1}{p}$ Gradient of tangent at Q=$\frac{1}{p}$ Equation of tangent at P $\frac{y-2aP}{x-9p2}=\frac{1}{p}$ x-py+ap2=0……….1 Equation of tangent at Q. x-qy+aq^2=0………..2 solving equations 1 and 2 -py+qy+aq^2-aq^2 =0 (q-p)y+a(p-q)(p-q) =0 Y=a(p+q) Substitute for y in equation 1 X=Pa(P+q)-ap^2 =apq. R is (apq, a(P+q) If R lies on 2x+a=0 Then R satisfies thus equations. 2(apq)+a=0 2pq+1=0 Pq - $\frac{1}{2}$		

Mid-point of PQ is $m\left(\frac{ap^2+aq^2}{2},\frac{2ap+2aq}{2}\right)$. M $\frac{9}{2}$(p^2+q^2), 9(p+q) At M: X=$\frac{9}{2}$(p^2+q^2), y=a(p+q) p^2+q^2 =$\frac{2x}{9}$……………. 3 P+q = $\frac{y}{9}$……………… 4 From 3 $\frac{2x}{9}$= (p+q)2-2pq Pq =- $\frac{1}{2}$ $\frac{2x}{9}$ = (P+q) ^{2}+1…….. 5 Substitution equation 4 in 5 $\frac{2x}{9}$ =($\frac{y}{9}$)^2+1 2ax =y^2+a^2 Y^2=2ax-a^2		

13. (a) $\sqrt{x}-3+\sqrt{2x+1}=\sqrt{3x+4}$ Squaring both sides. ((x-3)+2$\sqrt{(x-3)(2x+1)}$+(2x+1)=3x+4 ⇒$\sqrt{(x-3)(2x+1)}$=3. 2x2-5x-12=0 (x+4)(2x+3)=0 X=4 or x=$\frac{-3}{4}$ Checking. When x=4 , LHS=RHS When x=$\frac{-3}{2}$, LHS=RHS X=4 is only solution.		
13(b). Let the first term of an AP be a and the common difference be d let also the first term of a GP be b and the common ratio be r. AP= a+(a+d)+(a+2d)+……….+(a+(n-1)d) G.P= b+br+br^2+br^3+……+ br^{n-1} ⇒Sum of the first terms; a+b=57………………i ⇒Sum of second terms. a+d+br = 94 at r=2. A+d+2b=94………………..ii Sum of the third term a+2d+br2=171 at r=2. a+2d+4b=171………………iii		

14. (a) let y= $\frac{2x^2}{x^2+1}$

$$\text{Dy} = \frac{2(x+h)^2}{\left[(x+h)^2+1\right]} - \frac{2x^2}{x^2+1}$$

$$\frac{2(x+h)^2}{\left[(x+h)^2+1\right]} - \frac{(x^2+1)-2x^2\left[(x+h)^2\right]+1}{\left[x^2+1\right]}$$

$$\frac{\left[2\left(x^2 2xh+h^2\right)+1\right]\left[x^2+1\right]-2^2\left[x^2+2xh+h^{2+1}\right]}{\left[(x+h)^2+1\right]\left[x^2+1\right]}$$

$$\frac{\left[2x^2+4xh+2h^2+1\right]\left[x^2+1\right]-2x^2\left[x^2+2xh+h^2+1\right]}{\left[(x+h)^2+1\right]\left[x^2+1\right]}$$

$$\frac{2x^4+2x^2+4x^3h+4xh+2x^2h^2+2h^2-2x4-4x^3h-2x^2h^2}{\left[(x+h)^2+1\right]\left[x^2+1\right]}$$

$$\frac{4xh+2h^2}{\left[(x+h)^2+1\right]\left[x^2+1\right]}$$

$$\frac{dy}{dx} = \frac{4x+2h}{\left[(x+h)^2+1\right]\left[x^2+1\right]}$$

$$\frac{dy}{dx} = \frac{4x}{(x^2+1)(x^2+1)}$$

$$\frac{dy}{dx} = \frac{4x}{(x^2+1)^2}$$

(b) $X^2+6x+34=(x+3)^2+25$ $25\left[1+\left(\frac{x+3}{5}\right)^2\right]$ Let $\frac{x+3}{5}=\tan\theta$ $\frac{1}{5}=\sec^2\theta\frac{dQ}{DX}$ When x=3 ,tanΘ=0 Θ=tan^{-1}(0) = 0 X=2 tanΘ=$\frac{5}{5}$ Θ = tan^{-1}(1) =$\frac{\pi}{4}$ $\int_{-3}^{2}\frac{dx}{x^2+6x+34}=\int_0^{\pi/4}\frac{1}{25}\left(\frac{1}{1+\tan^2\theta}\right)5\sec^2\theta dx$ $=\frac{1}{5}\int_0^{\pi/4}d\theta$ $=\frac{1}{5}[\theta]_0^{\pi/4}$ $=\frac{\pi}{20}$		
15. (a) $\frac{dy}{dx}$ = y+ $\tan\left(\frac{y}{x}\right)$ using y=ux From y=ux $\frac{dy}{dx}$=u$\frac{d(x)}{dx}+x\frac{d(u)}{dx}$ $\frac{dy}{dx}=u+x\frac{du}{dx}$ $\left(u+x\frac{du}{dx}\right)$= ux +tan(u) xu+x$^2\frac{dy}{dx}dx=x4+\tan u$		

$x^2\frac{du}{dx}=\tan u$ separating variables X^{2}du = tan dx $\frac{du}{\tan u}=\frac{dx}{x^2}$ $\frac{1}{\tan u}du=\frac{dx}{x^2}$ $\frac{\cos u}{\sin u}du=x^{-2dx}$ $\Rightarrow\int\frac{\cos u du}{\sin u}=\int^{x^2dx}$	

In sin u $=\frac{-1}{x}$+c But from y=ux $u = {}^{y}\!/\!_{x}$ $\Rightarrow$ in sin$\left({}^{y}\!/\!_{x}\right) = \frac{-1}{x} + c$ Or $In\cos {}^{y}\!/\!_{x} + \frac{1}{x} + A = 0$	
(b) Let A represent the number of accidents and P represent the number of police deployed. $\frac{dA}{dp} = -kp$. dA=-Kpdp. SdA =-KSpdp. A=-kSpdp A=$\frac{-kp^2}{2} + c$ At A=7,P=2. 7=-k$\frac{4}{2}$+c. 7 =-2k+c……………. 1 At a=1 , P=4 1=-k$\frac{k16}{2} + c$ 1 =-8k+c…………11 K=1, c=9.	

So the equation connecting A and P is A=$\frac{-p^2}{2} + 9$ A= $\frac{-p^2}{2} + 9$ 2A= -p^2+(9X2) P^2 = (2X9)-2A P2=18-2A.	
(i) When there is no policeman P=0. P2 = 18-2A O = 18-2A $\frac{2A}{2} = \frac{18}{2}$ A = 9	

There are 9 accidents if no policeman is deployed.		
(ii) To completely stump out accidents ie A=0 From P2= 18-2A. A=o P2=18 P=√18 = 4.24 =5 5 policemen are required.		
16. $k=\frac{5t}{16+\left(t/a\right)^2}$ $k=\frac{5t}{1+t^2/a^2}$ $k=\frac{5a^2t}{a^2+t^2}$ $\frac{dk}{dt}=\frac{\left(a^2+t^2\right)-5a^2-5a^2t\left(2t\right)}{\left(a^2+t^2\right)^2}$ But at maximum concentration $\frac{dk}{dt}$=0 $5t^2a^2+5a^4-10a^2t^2=0$ At t=6 $180a^2+5a^4-360a^2=0$ $5a^2(a^2-36)=0$ Either 5a2=0 , a=0 Or a2-36=o, a=± 6 A=± 6		
(b) Volume of a cone. $\frac{1}{3}\pi r^2 h]$ $v=\frac{1}{3\sqrt{3}}\pi r^3$ Given $\frac{dv}{dv}=\frac{\pi r^2}{\sqrt{3}}$ Required $\frac{dv}{dt}=\frac{dv}{dv}\,\frac{dv}{dt}$ $\frac{dr}{dt}=\frac{\sqrt{3}}{\pi}.\frac{dv}{t}$		

From $\frac{dv}{dt}$=9m/s When v=$\frac{6}{60}$ t=1 $v=\frac{6}{60}.20\sqrt{3}$ t=$20\sqrt{3}$ V=$3\sqrt{3}$ From V=$\frac{1}{3\sqrt{3}}\pi r^3$ $3\sqrt{3}=\frac{1}{3\sqrt{3}}\pi r^3$ $\pi r^3=27$ $r=\frac{3}{\pi^{\frac{1}{3}}}$ $\frac{dr}{dt}=\frac{\sqrt{3}}{\pi\left(\frac{3}{\pi^{1/3}}\right)^2}\frac{dv}{dt}$ $\frac{dr}{dt}=\frac{\sqrt{3}}{\pi^{1/3}}$		

P425/1
PURE
MATHEMATICS
Paper 1
Nov 2020
3hrs

ST. MARYS' KITENDE

Uganda Advanced Certificate of Education

RESOURCEFUL MOCK EXAMINATIONS 2020

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Attempt ***all*** *the* ***eight*** *questions in* ***Section A*** *and****Not*** *more than* ***five*** *from* ***Section B.***

Any additional question(s) will not be marked.

All working must be shown clearly.

Silent non-programmabe calculators and mathematical tables with a list of formulae may be used.

Graph papers are provided.

SECTION A: (40MARKS)

*Answer **all** the **eight** questions in this Section.*

1. Solve the simultaneous equations; $\frac{1}{2y}+\frac{1}{x}=4$; $\frac{3}{x}-\frac{1}{y}=7$. (5marks)

2. Prove that; $\frac{log_2 x - log_2 x^2}{log_4 x^3}+\frac{5}{3}=log10$. (5marks)

3. Given the parabola $y^2=8x$,
a) Express a point T on the parabola in parametric form using t as the parameter. (2marks)
b) If parameter r gives point R, show that the gradient of chord TR is $\frac{2}{t+r}$. (3marks)

4. Find $\int x^3 e^{x^2} dx$. (5marks)

5. The line $r=\begin{pmatrix}1\\2\\1\end{pmatrix}+\mu\begin{pmatrix}2\\a\\b\end{pmatrix}$ meets a plane P perpendicularly at the point $(3,1,2)$.
Find the vector equation of the plane. (5marks)

6. Solve $\sin(120^0+3x)=cos(90^0-x)\ for\ 0^0\le x\le 90^0$. (5marks)

7. A roll of fencing material $152m$ long is used to enclose a rectangular area using two existing perpendicular walls. Find the maximum area enclosed. (5marks)

8. Solve the differential equation $\frac{dy}{dx}x-x=y$ given that $y=e$ when $x=e$. (5marks)

SECTION B : (60MARKS)

9. a) Prove that; ${}^{n+1}_{r+1}C+{}^{n+1}_{r+2}C={}^{n+2}_{n-r}C$. (6marks)

b) Two blue, three red and four black beads are to be arranged on a circular ring made of a wire so that the red are separated. Find the number of different arrangements. (6marks)

10. Given that; $f(x)=\frac{1+2x}{1-x}$
a) Find Maclaurin's expansion of $f(x)$ upto the term in x^3. (8marks)
b) Hence, find the value of $\frac{1.02}{0.99}$ to four significant figures. (4marks)

11. a) Given that; $ysinx + xcosy = \frac{\pi}{2}$, find $\frac{dy}{dx}$. (4marks)

b) A square prism is always three times the width in length. If the volume increases at a constant rate of $4cm^3s^{-1}$, find the rate of change of the cross-sectional area when the width is $12cm$. (8marks)

12.

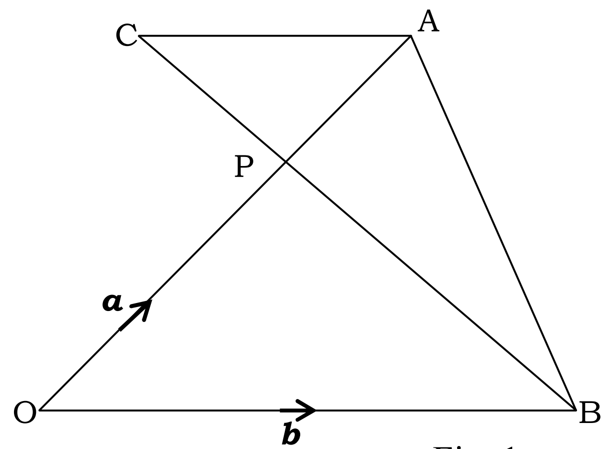


Fig. 1

Figure 1 shows points A and B with position vectors a and b respectively. $3AC = BO$.

a) Express each of the following in terms of vectors $a$and b.

i) BA (2marks)

ii) BC (3marks)

b) Find the ration $BP:PC$ (7marks)

13. a) Prove that $cos(tan^{-1}x) = (x^2+1)^{-\frac{1}{2}}$. (4marks)

b) i) Prove that $\frac{cos^2 4x + cos4x + sin^2 4x}{cos^2 4x - cos4x + si\ \ ^2 4x} = 3$ for $0 \leq x \leq \pi$. (4marks)

14. The lines L_1 and L_2 are perpendicular and intersect at $P(0,5)$. Line L_1 meets the x-axis in the first quadrant at Q such that $PQ = 13$ units. If L_2 meets the x-axis at R, without graphical construction, find the area of the triangle PQR. (12marks)

15. Given that $Z1 = 2 - 3i, Z_2 = 1 + 2i\ and\ Z_3 = 3 - 4i$.

a) Express $\frac{Z_1+Z_2}{Z_1Z_2}$ in the form $a + bi$ where a and b are real numbers. (6marks)

b) Find a polynomial $p(x)$ of degree four where the roots of $p(x) = 0$ are Z_2 and Z_3. (6marks)

16. Evaluate; $\int_2^3 \frac{x^4-x^3-x^2+4x-1}{(x-1)(x^2+1)}dx$. (12marks)

END

P425/1
PURE MATHEMATICS
Paper 1
TUESDAY, 7th August 2018 (Morning)
3 hours

ACHOLI SECONDARY SCHOOLS EXAMINATIONS COMMITTEE

Uganda Advanced Certificate of Education

Joint Mock Examinations, 2018

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- ✓ Answer ALL the EIGHT questions in section A and any FIVE questions in section B.
- ✓ All necessary working MUST be shown clearly.
- ✓ Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 marks)

Answer ALL the questions in this section. All questions carry equal marks.

1. A curve is defined by the parametric equations: $x = t^2$, $y = \frac{1}{t}$ $(t \neq 0)$. Find the equation of the tangent to the curve at the point where the curve cuts the x-axis. (05 marks)

2. If $z = 2 + i$ is a root of the equation $2z^3 - 9z^2 + 14z - 5 = 0$, find the other roots. (05 marks)

3. (i) Find the binomial expression of $\frac{1}{(a + bx)^2}$ up to and including the term x^3.

(ii) Given that the coefficient of the x term is equal to the coefficient of the x^2 term, show that $3b + 2a = 0$. (05 marks)

4. Find the coordinates of the point C on the line joining the points A(–1, 2) and B(–9, 14) which divides AB internally in the ratio 1 : 3. Find also the equation of the line through C which is perpendicular to AB. (05 marks)

5. Solve the equation $5 \text{ Cos } \theta - 3 \text{ Sin } \theta = 4$ for $0^{\circ} \leq \theta \leq 360^{\circ}$. (05 marks)

6. Evaluate $\int_2^5 x\sqrt{(x - 1)}\,dx$

(05 marks)

7. Find the Cartesian equation of the plane passing through the midpoint of AB with A(–1, 2, –5) and B(3, 0, –1) which is perpendicular to the line $\frac{x - 1}{2} = \frac{y + 7}{-3} = \frac{6 - z}{8}$. (05 marks)

8. Solve the equation: $\frac{dy}{dx} - y \tan x = \text{Cos } x$, given $y = 0$ at $x = \frac{\text{®}}{2}$. (05 marks)

SECTION B (60 marks)

Answer only FIVE questions from this section. All questions carry equal marks.

Question 9:

(a)Evaluate $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\left(5x^3 - 12x + 4\right)}{\sqrt{\left(1 - x^2\right)}}\,dx$ (05 marks)

(b) By means of the substitution t = tan x, prove that $\int_0^{\text{®}/4} \frac{dx}{1 + \sin 2x} = \frac{1}{2}$ and find the value of $\int_0^{\text{®}/4} \frac{dx}{\left(1 + \sin 2x\right)^2}$. (07 marks)

Question 10:

(a) If z_1 and z_2 are complex numbers, solve the simultaneous equations:

$4z_1 + 3z_2 = 23$

$z_1 + iz_2 = 6 + 8i$, giving your answers in the form $x + iy$. (06 marks)

(b) Find the value of the complex number z given that $z^3 = \frac{5 + i}{2 + 3i}$. (06 marks)

Question 11:

(a) Find the angle between line $\frac{x - 2}{4} = \frac{y}{3} = \frac{z - 1}{2}$ and the plane $-3x + 5y + 6z = 10$. (04 marks)

(b) A plane P_1 passing through the points (1, –1, 0) and (1, 0, –3) is perpendicular to the plane P_2 having equation: $x + y = 6z = 0$. Find:

(i) the equation of P_1

(ii) the angle between P_1 and another plane P_3 with equation: $x - y + z = 7$.

(08 marks)

Question 12:

A disease is spreading at a rate proportional to the product of the number of people already infected and those who have not yet been infected. Assuming that the total number of people exposed to the disease is N;

(a) Write down a differential equation.

(b) Initially 20% of the population is infected. Two months later 40% of the population is infected. Determine how long it takes for only 25% of the population to remain uninfected. (12 marks)

Question 13:

(a) Determine the equation of the circle passing through the points A(–1, 2), B(2, 4) and C(0,4). (06 marks)

(b) If $y = mx - 5$ is a tangent to the circle $x^2 + y^2 = 9$, find the possible values of m. (06 marks)

Question 14:

Sketch the curve $y = \frac{x + 1}{(x - 1)(2x + 1)}$, showing clearly the asymptotes and turning points. (12 marks)

Question 15:

(a) Determine the maximum value of the expression: 6 Sin x – 3 Cos x. (03 marks)

(b) Prove that $\frac{\text{Cos}\,11° + \text{Sin}\,11°}{\text{Cos}\,11° - \text{Sin}\,11°} = \tan 56°$ (03 marks)

(c) In a triangle ABC, prove that $\text{Sin B} + \text{Sin C} - \text{Sin A} = 4\,\text{Cos}\frac{A}{2}\,\text{Sin}\frac{B}{2}\,\text{Sin}\frac{C}{2}$ (06 marks)

Question 16:

(a) Using Maclaurin's theorem, expand e^{-x}Sin x up to the term in x^3. Hence, evaluate $e^{-\text{®}/3}\,\text{Sin}\frac{\text{®}}{3}$ to 4 significant figures. (05 marks)

(b) The curve $y = x^3 + 8$ cuts the x and y axes at the points A and B respectively. The line AB meets the curve again at point C. Find the coordinates of A, B, C and hence, find the area enclosed between the curve and the line. (07 marks)

THE END

S.6 P425/1 QUESTIONS 2020 (STAHIZA)

1. (a) Prove that the number $a_n = 4^n + 5$ is divisible by 3 for all positive integral values of n.

   (b) The sum of **n** terms of the progression in $\mathbf{S_n = 2n^2+3}$.Find the tenth term.

   (c) Prove by induction that.

   (i) $\sum_{r=1}^{n} 3^{n-1} = \frac{1}{2}(3^n-1)$

   (ii) $\sum_{r=1}^{n} \frac{1}{(2r+1)(2r+3)} = \frac{n}{3(2n+3)}$

   iii) $\sum_{r=1}^{n} Sin\, r\theta = \frac{Sin(\frac{n+1}{2})\theta Sin(\frac{n\theta}{2})}{Sin\frac{\theta}{2}}$

   (d) DON operate an account with Cairo bank which pays a compound interest rate of 12.5% per annum. They opened the account at the beginning of the year with Shs800,000 and deposit the same amount of money at the beginning of every year. Calculate how much they will receive at the end of 10 years. After how long will the money have accumulated to 3.32 million.

2. (a) A polynomial P(x) is a multiple of x – 3 and the remainders when P(x) is divided by x + 2 and x-5 are 6 and-7 resp. Find the remainder when P(x) is divided by $x^3 - 5x^2 - 4x + 20$.

   (b) When $P(x) = x^3 + ax^2 + bx + c$ is divided by $x^2 - 4$, the remainder is 2x + 11. Given that x + 1 is a factor of P(x), Find the values of a, b and c.

   (c) If the function $P(x) = x^4+ax^3+bx^2+6x-5$ is divisible by$(x-2)^2$.

   Find the values of a and b.

   (d) Give that $x^4 - 6x^3 + 10x^2 + ax + b$ is a perfect square, find a and b.

   The roots of the equation $ax^2+bx+c=0$ where a,b and c are non Zero constants are α and β,and the roots of equation $ax^2+2bx+c=0$ are θ and μ. Find the equation whose roots are $\alpha\theta+\beta\mu$ and $\alpha\mu+\beta\theta$.

3. (a) Express in partial fractions; $f(x) = \frac{x^2-x}{(x^2+3)(x^2+2)}$Hence $\int f(x)dx$

   (b) $\int \frac{x^6-x^5-4x^2+x}{x^4+3x^2+2}\,dx$

   (c) Express $f(x) = \frac{4x+5}{(x+1)(2x+3)}$ in partial fractions; hence find $f'(x)$ and $f''(x)$.

d) Express y= $\frac{2x^2+3x+5}{(x+1)(x^2+3)}$ in partial fractions and hence show that $\frac{dy}{dx} = -\frac{2}{3}$ when x=0 ,and evaluate $\int_0^{\sqrt{3}} ydx$. state the mean value of y in the interval 0≤x≤$\sqrt{3}$

4. Differentiate;

(a) $\sqrt[3]{\frac{x^2-2}{x^2+4}}$ (b) $\sqrt{\frac{Sin\ 2x}{Cos^3x}}$ (d) $e^{-3x}sin^2 2xcos3x$ (e) $\frac{e^{2x}tanx}{(x^2-2)sinxcosx}$

(c) (i) If y = t^2 - cos t and x = sin t, find $\frac{d^2y}{dx^2}$. (ii)Find $\frac{dy}{dx}$ of **tan4x⁰** from1st principles.

5. Integrate the following;

(a) $\int_1^3 \frac{2x^2+3x}{2x-1}$dx (b) $\int \frac{x+1}{125x^3-8}$dx (c) $\int \frac{1}{(1+\sqrt{x})\sqrt{x}}$dx

(d) $\int_0^{\Pi/2} Sin^3xCos^2x$ dx (e) $\int_0^{\Pi/2} Sin\ 2x\ Sin\ 5x$ dx

(f) Show that $\int_0^{\Pi/4}(1+tanx)^2$ dx = 1 + $ln2$ (g) $\int \frac{1}{x^4-9}dx$

(g) Show that $\int_0^1 \frac{1}{(x+1)(x^2+2x+2)}$dx = $\frac{1}{2}ln\frac{8}{5}$

(h) Show that $\int_0^{\Pi/2} \frac{Sin^3x}{2-Sin^2x}$dx = $\frac{\Pi}{2}$ - 1

(i) Show that $\int_0^{\Pi/2} \frac{4}{3+5cos\theta}$ dθ = ln 3

(j) $\int_0^{\Pi/4} \frac{1}{5Cos^2\theta-1}$ dθ (k) $\int_0^3 \frac{x+3}{x^2+3}$ dx

(l) $\int xlnx\ dx$ (m) $\int xtan^2 x$ dx (n) $\int x^3\ Sec\ x^2$ dx

(o) $\int \frac{6(x-3)}{x(x^2+9)}$ dx (p) $\int \frac{e^x+e^{2x}}{1+e^{2x}}$dx q) $\int_0^2 \frac{x^4+2x}{(x-1)(x^3-1)}\ dx$

(r) Evaluate $\int_0^{\frac{\pi}{6}} secxdx$ using **sinx=u**.Hence show that $\int_0^{\frac{\pi}{6}} sec^3xdx=\frac{1}{3}+\frac{1}{4}ln3$

6. (a) If y = e^xsinx, Show that $\frac{d^2y}{dx^2}$ = 2($\frac{dy}{dx}$ - y). By further differentiation of this result, find the Maclaurin's expansion as far as the term in x^6 .

(b) Show that using maclaurins theorem,ln$\sqrt{\frac{1+x}{1-x}}$ =x+$\frac{x^3}{3}$ +$\frac{x^5}{5}$ +$\frac{x^7}{7}$.

(c) Show that if y = $e^{3x}sin4x$, $\frac{d^2y}{d^2x} - 6\frac{dy}{dx} + 25y$ =0.

7. (a) Expand $(8+3x)^{1/3}$ in ascending powers of x as far as the term in x^3, stating the values of x for which the expansion of x is valid. Hence obtain an approximate value for $\sqrt[3]{8.72}$

(b) Expand $(1+\ 16x^2)^{1/2}$ in descending powers of x in the term as far as the third term.

(c) Prove that if x is so small that its cube and other higher powers are neglected,then

$\sqrt{\frac{1+x}{1-x}}$ =1+x+$\frac{1}{2}x^{2}$. By taking x=$\frac{1}{16}$,Prove that $\sqrt{17}$ =4$\frac{33}{128}$

8 (a) According to Newton's law, the rate of cooling of a body in air is proportional to the difference between the temperature T of the body and T_o of the air. If the air temp is kept constant at 20^0c and the body cools from 100^0Cto 60^0C in 20 minutes, in what further time will the body cool to 30°C.

(b) A liquid is being heated in an oven maintained at constant temperature of 180°C. It is assumed that the rate of increase in temperature of the liquid is proportional to (180 – θ), where θ°C is the temperature of the liquid at time t minutes. If the temperature of the liquid rises from 0°C to 120°C in 5 minutes, find the temperature of the liquid after a further 5 minutes.

(c) The rate of speed of bush fire spreading is proportional to the area of unburnt

Bush .At a certain moment 0.6 of the bush had been burnt, 2hrs later 0.65 of

The bush was bunrt.Find the fraction remaining unburnt after 5hours.

9. (a) (i) If Z_1 = 6(Cos $\frac{5}{12}\Pi$ +i Sin$\frac{5}{12}\Pi$) and Z_2 = 3($Cos\frac{1}{4}\Pi$ +i Sin $\frac{1}{4}\Pi$). Find Z_1Z_2 and $\frac{Z_1}{Z_2}$ in the form $a+ib$.

(ii) Express each of the complex numbers in a polar form if

$Z_1=\frac{(1-\sqrt{3}\ i)^4\ (2+2\sqrt{3}\ i)^5}{(3i-4)^3}$ and $Z_2 = \left(cos\frac{\pi}{3}+isin\frac{\pi}{3}\right)^6$

(b) Find the locus of the complex number $|Z-1-i|<3$, represent it on the Argand diagram and shade the region representing it.

(c) (i) Solve the equations $Z^3 - 8i = 0$ and $z^3+i=0$.

(ii) Find in the form $a+ib$, the roots of the equation $Z^3 - 26 - 18i = 0$

(d) If z is a complex number,describe and illustrate on the argand diagram this locus

Given by each; (i) $\left|\frac{z-2i}{z-i}\right|$=2 (ii) $\arg(z+2+3i) = \frac{\pi}{3}$.

(e) Show that the locus of the complex number Z of $|z+1|=2|z-1|$ is an ellipse and Find its equation,hence find its focus,eccentricity and equation of directrix.

10 Solve the following by row reduction to echelon form.

(b) $3x - 2y + 4z = -7$,$x + y - 6z = -10$ and $2x + 3y + 2z = 3$

(c) By using the method of synthetic division, divide

$6x^4 - 17x^3 + 22x - 9$ by 2x -3

(d) Find the equation of circle that touches line y=x at pt (3,3) and passes thru B(5,9).

(e) Solve the simultaneous equations

$2^x + 4^y = 12$ and $3(2^x) - 2(2^{2y}) = 16$ .Hence show that $4^x + 4(3^{2y}) = 100$

(f) When the quadratic expression ap^2+ bp + c is divided by p-1, p-2 and p+1, the remainders are 1, 1 and 25 respectively. Determine the factors of the expression.

11. (a) Find the term independent of x in the expansion of $(2x - \frac{1}{x^2})^{12}$

(b) Expand $(\frac{1+x}{1+3x})^{1/3}$ up to the term. Hence by taking x = $1/125$, use your result to calculate the cube root of 63 correct to four decimal places.

12. (a) Show that $\cos^{-1}\left(\frac{63}{65}\right) + 2\tan^{-1}\left(\frac{1}{5}\right) = \sin^{-1}\left(\frac{3}{5}\right)$

(b) Solve the equation $(i) Sin\ 3x + {}^{1}/_{2} = 2\ Cos^{2}$ x for 0°≤ x ≤ 360°

(ii) 4sinxcos2xsin3x=1 for 0° ≤x≤180°

(c) Show that in any triangle A B C, $\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\cot\left(\frac{A}{2}\right)$. Then solve the triangle two sides 5 and 7 and the included angle of 45°.

(d) Show that for allvalues of $\theta, cos\theta + \cos\left(\theta + \frac{2}{3}\pi\right) + \cos\left(\theta + \frac{4}{3}\pi\right) = 0.$

Hence show that $cos^2\theta + cos^2\left(\theta + \frac{2}{3}\pi\right) + c0s^2\left(\theta + \frac{4}{3}\pi\right) = \frac{3}{2}.$

(e) (i) Prove that $4\cos\theta 3\cos\theta + 1 = \frac{\sin 5\theta}{sin\theta}$

(ii) In any triangle ABC,Prove that TanBcotC= $\frac{a^2+b^2-c^2}{a^2-b^2+c^2}$.

13. (a) Solve the inequality,

(i) $\frac{x-1}{x-2} > \frac{x-2}{x+3}$ (ii) $2|x-1| = |x+3|$ (iii) $|5x-6| = x^2$

(b) Sketch the curve $f(x) = \frac{x^2+2x+3}{x^2+3x+2}$.

(c) If $y = \frac{(x-1)^2}{(x+1)(x-3)}$. Find the values of y for which the curve is not defined,

Hence find the nature of the turning points. sketch the curve.

(d) Find the co-ordinate of intersection of $y = \frac{x}{x^2+1}$ and $y = \frac{x}{x+3}$.

Sketch both curves on Same axes and show that the area of finite region in the first quadrant enclosed by two curves is $\frac{7}{2}ln5$ -3ln3-2.

(e) The domain of the function defined by $f(x) = \frac{4(2x-7)}{(x-3)(x+1)}$ is the set of all real Values of x other than 3 and -1.Express f(x) in partial fractions,hence or Otherwise,show that $f^1(x)$ is zero for two +ve integral values of x.Find the Turning points of this function.Hence sketch f(x).Dertermine the area of the Region bounded by the cyrve,the x-axis and the lines x=4,x=6.

(Leave your answer in logarithmic form.)

14. (a) Show that at any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ may be written as (a sec θ, b tan θ)

(b) Find c in terms of a, b , m if y = m x + c is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ .Find the asymptotes.

(c) The normal at the point P(5 Cosθ,4sinθ) on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the X and Y axes at Land M respectively. Show that the locus of R ,the midpoint of LM is an ellipse having the same eccentricity as the given ellipse.

(d) The curve $b^2x^2 + a^2y^2 = a^2b^2$intersects the positive X-axis at A and the Y-axis at B.

(i) Determine the equation of the perpendicular bisector of AB.

(ii) Given that this line intersects the X-axis at P and that M is the bisection of AB.Show that the area of triangle PMA is$\frac{b(a^2+b^2)}{8a}$

(e) (i) State the vertex,focus and directix of **$8y^2 + 6y - 9 = 4x$** and **$2x^2+3y^2-4x+9y+5=0$.**

(ii) Show that the two tangents of gradients m to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are

Y= mx $\pm \sqrt{(a^2m^2+b^2)}$.find eqnsof tangents to ellipse $\frac{x^2}{6}+\frac{y^2}{3}=1$ at (-2,5).

15. (a) Find the angle between $\frac{x-3}{2}=\frac{2y-1}{4}=\frac{3z-1}{4}$ and $-4x+3y+2z=7$

(b) Find the angle between the line of intersection of the planes 2x + y + 3z = 4 and 3x + 2y + 2z = 7 and the line $\frac{1-x}{1}=\frac{y-2}{2}=\frac{z-3}{4}$

(c) Find the distance between the planes 2x – 3y + 4z = 7 and 8x – 12y + 16z = 6.

(d) Find the equation of the plane through the (1,0,-1) and containing the

line $X=-y=\frac{z}{2}$.

(e) Find the equation of the plane containing the lines $\frac{x-3}{5}=\frac{y+1}{2}=\frac{z-3}{1}$ *and*

$\frac{3-x}{-2}=\frac{y+1}{4}=\frac{z-3}{3}$.

(f) Find the equation of the plane through the point (1,-2,1) which also

Contains the line of intersection of the planes x+y+z+6=0 and

x-y+z+5=0.

(g) Show that vectors $i-2k, -2i+j+3k$ and –i+j+k form a triangle .

Hence or otherwise find the area of the triangle.

(h) Show that the vectors **a=3i+j-4k,b=4i-j-3k,c=5i-3j-2k** are coplanar.

(I) The equations $\frac{x-3}{2}=\frac{y-5}{1}=\frac{z-2}{-3}$ *and* $\frac{x+1}{3}=\frac{y+4}{1}=\frac{z-2}{-2}$ represent pipes

A and B in a chemical plant. Find the the length of the shortest pipe that

can be fitted at its end points.

- ***TO THE PROBLEMS OF YOUR LIFE,YOU'RE THE SOLUTION ,AND TO THE THE QUESTIONS OF YOUR LIFE ,YOU'RE THE SOLUTION.***
- ***IF YOU ARE GOING TO ACHIEVE EXCELLENCE IN BIG THINGS, YOU DEVELOP THE HABIT IN LITTLE MATTERS.***
- ***WHEN SPIDER WEBS UNITE, THEY CAN TIE UP A LION.***
- ***PRAYER AND HARD WORK GO TOGETHER WITH DISCPLINE.***
- ***THINGS DO NOT CHANGE; WE CHANGE***

GOOD LUCK AND SUCCES

SMATA

4TH ANNUAL POST MOCK SEMINAR 2023

S.6 SUBSIDIARY MATHEMATICS

AT

ST. JOSEPH OF NAZARETH HIGH SCHOOL

24th September 2023

SEMINAR QUESTIONS

PART I

PURE MATH QUESTIONS

1. Solve the differential equation $\frac{dy}{dx} = \frac{7x^2+1}{8y}$; given that y =2 when x =0
2. Evaluate i). $\int_1^2 (3x^2 - 4x + 2)\, dx$

   ii). $\int_1^4 \left(\sqrt{x} - \frac{4}{\sqrt{x}} + 2\right) dx$

3. Given that $\frac{1}{\sqrt{2}} - \frac{\sqrt{2}+1}{1+3\sqrt{2}} = a\sqrt{2} + b$ where a and b are constants, find the values of a and b.

4. a). Solve the equation $2\sin 2\theta = 3\cos\theta$, for $0^0 \leq \theta \leq 360^0$

   b). Without using tables or a calculator, find in surd form the value of tan 105^0.

   c). If $Sin\, x = a$, find in terms of a, i). $cosx$ ii). $cosecx$.

5. Solve the equation $3(1-x)^2 + 7(1-x) + 4 = 0$

6. If $\frac{1}{\alpha}$ and $\frac{1}{\beta}$are the roots of the equation$4x^2 - 8x + 1 = 0$, find the equation whose roots are $\alpha \;\; and\, \beta$.

7. In a geometric progression, the second term exceeds the first term by 20 and the fourth term exceeds the second by 15. Find the possible values of the first term.

8. An Arithmetic Progression (A.P) has a common difference of -3. The sum of the first twenty terms is ten times the second term. Find the sum of the first 15 terms of the A.P.

9. Given that x+1 is a factor of the polynomial $2x^3 + 5x^2 + x - 2$, find the other two factors.

10. The points P,Q,and R have position vectors $2i + 2j$, $i + 6j$ and $-7i + 4j$respectively.
    a).i). Find the vector QR and PQ
    ii). Show that triangle PQR is a right angled at Q.

    b). Find the angle between PR and PQ

11. a). Given that $A = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$, evaluate $det(A^2 - 2A)$
    b). Given the Matrix $N = \begin{pmatrix} a & a+1 \\ a-1 & a+2 \end{pmatrix}$. Find the value of a for which N is singular.

12. Solve the simultaneous equations below using matrix method

$$5x - 3y = 16$$

$$4y + x = -6$$

13. Two types of bread A and B are manufactured by a certain bakery. Type A requires 2kg of wheat flour, 1.5 litres of oil,3 eggs and 0.2kg of sugar. And type B requires 4kg of wheat flour, 2 litres of oil, 4 eggs and 0.4kgs of sugar. The cost of wheat flour is shs2500 per kg, oil is shs3000 per kg shs 350 per egg and shs 2800 per kg of sugar. If shop X ordered for 5 breads of

type A and 8 of type B while shop Y ordered for 7 of type A and 10 of type B.

i. Form three matrices from the information above

ii. Use matrix multiplication to find the cost of each type of bread

iii. Find the total cost required to work on the two orders

14. Polynomial $P(x) = x^4 + \boldsymbol{m}x^3 + \boldsymbol{n}x^2 + 5x + 3$ has a remainder of $2x + 1$ when divided by $x^2 + 3x + 2$. Find the values of **m** and **n.**

15. The population of a certain organism grows at a rate proportional to the number (N) of organisms present at time t . initially the number was 500 and it doubles every after 36 hours. Find the time for the number of the organisms to triple.

16. solve the differential equation $\frac{dy}{dx} = \frac{x}{1-y}$ when $y = 5$ and $x = 0$

PART II

APPLIED MATH TOPICAL AREAS

STATISTICS AND PROBALITY

DISCRETE RANDOM VARIABLE

17. A random variable x has a probability distribution given in the table below.

x	-1	0	1	2
$p(x = x)$	a	0.3	0.4	b

a) If the mean E(x) = 0.7. Find the values of a and b hence mode and median.

b) Determine;

i). variance and standard deviation

ii). $p(x \leq 0)$

iii). $E(3x - 1)$

BINORMIAL DISTRIBUTION

18.a). Given that $X \approx B(10, p)$ and variance $\frac{15}{8}$. Determine the possible value of p hence find $p(x \geq 7)$if P is less than 0.5.

b). In a certain family, the chance of having a baby boy is double that of a girl. A family plans to have five children. Determine the probability that;

i). Atleast two will be girls.

ii). None will be a boy.

CONTINOUS RANDOM VARIABLE

19. A random variable has a pdf given by

$$f(x) = \begin{cases} cx\ , x = 1,2,3,4 \\ c(8-x), \ \ x = 5,6,7 \end{cases}$$

Find i). the value of c

ii). $P(3 \leq x \leq 5)$

iii). $P(x \geq 2 | x \leq 6)$

20. a). The pdf of a continuous random variable is given by

$$f(x) = \begin{cases} k(x^2 - 1)\ , & 0 \leq x \leq 2 \\ & 0\ otherwise \end{cases}$$

Find i). value of k

ii). $p(x \leq 1.5)$

iii). Mean

iv). Variance

v). Median

21. 21. A continuous random variable is given by the pdf

$$f(x) = \begin{cases} ax, & 0 \leq x \leq 3 \\ 3a(4-x)\ , & 3 \leq x \leq 4 \\ 0, & \text{Otherwise} \end{cases}$$

Where a is a constant

i) Sketch the pdf, hence value the value of **a**

ii) Determine $P(1 \leq x \leq 3)$

NORMAL DISTRIBUTION

22.a). A random variable x is given as $X \sim N\ (500,900)$, find $P(x \geq 500)$

b). A certain type of sweet potato as a mass which is normally distributed with mean 1 Kg and standard deviation 0.15Kg sack loaded with potatoes. Find the probability that potato picked at random from the sack weighs;

i) More than 0.79Kg
ii) at most 1.13 Kg
iii) Between 0.85Kg and 1.15Kg
iv) Number of potatoes between 0.75Kg and 1.29 Kg if 10,000 potatoes were in a sack.

STATISTICS

23.a). The height in centimeters of 6 tree seedlings are 60, 55, 46, 43, 56 and 58. Using assumed mean of 50. Find the mean height of the seedlings hence variance.

24. Below are marks obtained by 50 candidates in a Sub-math test.

Marks	-<30	-<40	-<50	-<60	-<70	-<80
No. of Candidates	3	11	29	40	47	50

Draw a frequency distribution table and use it to determine:

i) Mean mark
ii) Standard deviation

c) Represent the data on an-ogive and use it to

i. Estimate the median mark
ii. Interquartile range
iii. Range of middle 50 percent mark

25. The following grades were obtained by 8 candidates in Mathematics and General paper.

Math (x)	A	O	B	F	E	C	D	B
G.P (y)	C3	D2	D1	P8	P8	D2	C3	D2

(a) Calculate the rank correlation coefficient for the grades.
(b) Comment on your results at 1% level of significance.

26. The table below shows the performance of Candidates in two mock examinations.

Mock 1	35	65	55	25	45	75	20	51	60	90
Mock 2	86	70	84	92	79	68	96	86	77	58

(a) Calculate the rank correlation coefficient between Mock 1 and Mock 2 performance and comment on your result at 5% significance level.

(b) Plot the data on the same graph and use it to comment on the performance in two mocks.

(c) Draw the best line of fit.

27. (a) The events A and B are such that P(A') = 0.3, P(B) = 0.1, P(A/B) =0.2

Find (i) P(AUB)

(ii) P(A/B')

(b). If P(A/B) = 0.2, P(A' $\cap B$) = 0.3 and P(B/A′) = 0.4, find

(i) P($A \cap B$)

(ii) P(A′ $\cup B$)

(c). If A and B are mutually exclusive events such that P(A) = 0.5 and $P(A \cup B) = 0.9$, find (i) $P(A' \cap B')$ (ii) $P(A' \cup B)$

28.a). A committee of 8 people in a village council is to be selected from 7 men and 6 women in how many ways can this be done if not more than 4 women are to be selected.

b). (i) How many arrangements can be made from the word ***MATHEMATICS***

ii). What is the probability of those arrangements when the A s are not together.

PRICE INDICES

29. The table below shows the prices in Ugshs of some food items in 2020, 2021 and 2022 and their corresponding weights.

Items	**Price (shs)**			**Weight**
	2020	**2021**	**2022**	
Matooke	15000	13000	18000	4
Meat	6500	6000	7150	1
Posho	2000	1800	1600	3
Beans	2200	2000	2860	2

Taking 2020 as the base year, calculate;

a) Simple aggregate price index for 2021 and comment on your results.
b) Price index for 2022 for each item.
c) Weighted aggregate price index for 2022
d) Simple price index for 2021.

30. Three items required by a certain family were Food(F), Water(W) and Shelter(S). The monthly expenditure on F, W and S were shillings 150,000, 108,000 and 120,000. If F and W were each twice as important as S. Using Shelter as the base price, determine the cost of living index hence comment.

MOVING AVERAGES

31. The table below shows the number of boxes of pens sold by a certain wholesale shop from the year 2019 to 2022.

Year	**Quarter**			
	1st	**2nd**	**3rd**	**4th**
2019	192	280	320	260
2020	300	360	380	270
2021	342	420	430	320
2022	424	480	510	412

a) Calculate the for-point moving averages for the data.
b) i). On the same axes plot the graphs of original data and the four-point moving averages.

ii). Comment on the number of boxes of pens sold over the four year period.

c) Use your graph to estimate the number of boxes to be sold in the first-quarter of 2023.

CURVE SKETCHING

32. Given the curve $y = 5 + 8x - 4x^2$; Determine,
    i). Turning points and the nature
    ii). Sketch the curve
    iii). Calculate the area bounded by the curve $y = 5 + 8x - 4x^2$ and the line $y = 5$

INEQUALITIES

33. Birungi has a maximum of shs 8000 to spend on bracelets. She will make two types of Bracelets. Type A which costs shs 1000 and type B which costs shs 800 each. Birungi plans to make more Bracelets of type B than type A. Also she wants atleast 2 bracelets of type A and over 6 bracelets of both types. Assuming she makes x bracelets of type A and y bracelets of type B.

    a) Write down four inequalities from the above information.
    b) Represent the inequalities on a graph taking 2cm to represent 1 unit on both axes.
    c) If Birungi gets a profit of shs 300 from each bracelet of type A and shs 200 from each bracelet of type B, find how many bracelets of each type she should make in order to realize maximum profits and state the maximum profit realized by Birungi.

Aim for this golden point at UNEB

Wishing you success

S.5 MATHEMATCIS PAPER ONE

CHAPTER ONE : DIFFERENTIATION

REVISION QUESTIONS 2020

Attempt all questions

1. Find the gradient function $\frac{dy}{dx}$ for each of the following functions.

   a) $y = x^2 + 7x - 4$

   b) $y = 6x^2 - 7x + 8$

   c) $y = 3x^6 - 7x^2 + 6x - 8$

   d) $y = 3x - \frac{5}{x} + \frac{6}{x^2}$

   e) $(x^7+2)(4x-1)$

2. Find the gradients of the following lines at the points indicated.

   a) $y=2x^3-x^2 + 3x - 1$ at (-1,-6)

   b) $y = x^2 + 7x - 4$ at (2,21)

   c) $y = 2x^2 - x + \frac{4}{x}$ at (2,8)

   d) $y = 3x + \frac{1}{x}$ at (1,4)

   e) $y = (x+1)(2x + 3)$ at (2,21)

3. If $f(x) = x^3 + 4x$ find

   a) $f(1)$

   b) $f'(x)$

   c) $f'(1)$

   d) $f''(x)$

e) $f''(2)$

4. If $f(x) = 3x^2 + \frac{24}{x}$ find

a) $f'(x)$

b) $f'(-12)$

5. Find the equation of the normal to the curve $y=x^2 + 4x-3$ at the point where the curve cuts the y- axis. Ans : $4y + x + 12 = 0$

6. Find the equation of the tangent to the curve $y=x^2-3x-4$ at the point where this curve cuts the line $x = 5$. Ans: $y = 7x-29$

7. Find the equation of the tangent to the curve $y = (2x-3)(x-1)$ at each of the points where this curve cuts the x- axis. Find the point of intersection of these tangents.

Ans: $y + x = 1$, $2y = 2x-3$;$(\frac{5}{4}, -\frac{1}{4})$

8. Find the equation of the normal to the curve $y=x^2-6x + 5$ at each of the points where the curve cuts the x- axis. Ans: $4y-x + 1 = 0$, $4y + x-5 = 0$

9. Find the equation of the tangent to the curve $y=x^2 + 5x-3$ at the points where the line $y = x + 2$ crosses the curve. Ans: $y = 7x-4$,$y + 5x + 28 = 0$

10. Find the coordinates of the point on the curve $y=2x^2$ at which the gradient is 8 Hence find the equation of the tangent to $y=2x^2$whose gradient is 8. Ans: (2 ,8),$y = 8x-8$

11. Find the coordinates of the point on the curve $3x^2-1$ at which the gradient is 3. Ans: $(\frac{1}{2}, -\frac{1}{4})$

12. Find the equation of the tangent to the curve $y = 2x^2-2x + 1$ which has a gradient of 0.5

Ans :$2y = x + 2$

13. Find the value of k for which $y = 2x + k$ is a tangent to the curve $y = 2x^2-3$. Ans $k = -\frac{7}{2}$

14. Find the equation of the tangent to the curve $y = (x-5)(2x + 1)$ which is parallel to the x-axis. Ans: $8y + 121 = 0$

15. A curve has the equation $y = x^3-px + q$. The tangent to the curve at the point (2,-8) is parallel to the x-axis. Find the values of p and q. find also the coordinates of the other point where the tangent is parallel to the x-axis. Ans p=12,q=8;(-2,24)

16. The function $ax^2 + bx + c$ has a gradient function $4x + 2$ and a stationary value of 1. Find the values of a, b and c. ***Ans a=2 , b=2 and c=$\frac{3}{2}$***

17. Find the second differential of y with respect to x for each of the following :

a) $y = 6x^2 + 7$

b) $y = 5x^3 + 6x - 5$

c) $y = 2 + \frac{3}{x}$

18. If $y = 3x^2 - x$ show that $y\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y + 1 = 6x$.

19. The tangent to the curve $y = ax^2 + bx + 2$ at $(1, \frac{1}{2})$ is parallel to the normal to the curve $y = x^2 + 6x + 10$ at (-2, 2). Find the values of a and b. Ans: 1 ,-2.5

20. Find the coordinates of any stationary points on the given curves and distinguish between them.

a) $y = 2x^2 - 8x$

b) $y = x^3 - x^2 - x + 7$

c) $y = 1 - 3x + x^3$

d) $y = (x-1)(x^2 - 6x + 2)$

e) $y = 18x - 20 - 3x^3$

f) $y = x^3 + 6x^2 + 12x + 12$

g) $y = x^3 - 3x^2 + 3x - 1$

21. Find the coordinates of the stationary points on the following curves and distinguish between them. Hence sketch the curves.

a) $y = x^4 + 2x^3$

b) $y = x^3 - 4x^2 + 4x$

c) $y = 5x^6 - 12x^5$

d) $y = x^4 - 4x^3 + 4x^2$

22. Differentiate $x^2 + \frac{1}{x}$ from first principles.

23. Differentiate $y = \frac{x}{x^2+1}$ with respect to x from first principles

24. Find the derivative of $\frac{1}{\sqrt{x}}$ from first principles

25. Find the equation of the normal to the curve $y = x^2 + 5x + 3$ that is parallel to the line

$y = 9x$.

26. Differentiate $P = x - x^2 + \frac{\pi}{2x}$ with respect to x where π is a constant

27. Find the equation of a tangent to the curve $y=2-4^{x2}+^{x3}$ at a point (1,-1)

28. Find the stationary points of the curve $y= 5+24x-9x^2-2x^3$ and distinguish the nature of these stationary points.

29. P and Q are neighboring points on the curve $y = 2(x-x^2)$. P is the point (x,y) and Q the point $(x + \delta x, y + \delta y)$. Find the value of the ratio $\frac{\delta y}{\delta x}$ and determine the gradient of the curve at point P.

30. Differentiate the following using the first principles.

a) $y = x^3 + x^2$

b) $y = \frac{1}{x^2}$

c) $y = \frac{1}{2x^2}$

31. P is the point (x,y) and Q the point $(x + \delta x, y + \delta y)$. On the graph of $y = \sqrt{x}$. Show that $\frac{\delta y}{\delta x} = \frac{1}{\sqrt{(x+\delta x)}+\sqrt{x}}$. And hence find the gradient of the curve at the point P.

32. Find the slope of the curve $y = ax^2 + bx + c$, where a,b and c are constants at the point whose x coordinate is x. At what point is the tangent to the curve parallel to the x- axis?

33. Find the gradient of the curve $y = 9x - x^3$ at the point where $x = 1$. Find the equation of the tangent to the curve at this point. Where does this tangent meet the line $y = x$?

34. Find the equation of the tangent at the point (2,4) to the curve $y = x^3 - 2x$. Also find the coordinates of the point where the tangent meets the curve again.

35. Find the equation of the tangent to the curve $y = x^3 - 9x^2 + 20x - 8$ at the point (1,4). At what points of the curve is the tangent parallel to the line $4x + y - 3 = 0$?

36. Find the equation of the tangent to the curve $y = x^3 + \frac{1}{2}x^2 + 1$ at the point $(-1, \frac{1}{2})$. Find the coordinates of another point on the curve where the tangent is parallel to that at the point $(-1, \frac{1}{2})$.

37. Find the points of intersection with the x-axis of the curve $y = x^3 - 3x^2 + 2x$,and find the equation of the tangent to the curve at each of these points.

38. Find the equations of the normals to the parabola $4y=x^2$ at the points (-2,1) and (-4,4). Show that the point of intersection of these two points lies on the parabola.

39. Find the equation of the tangent at the point (1,-1) to the curve $y = 2 - 4x^2 + x^3$. What are the coordinates of the point where the tangent meets the curve again? Find the equation of the tangent at this point.

40. Find the coordinates of the point P on the curve $8y=4-x^2$ at which the gradient is $\frac{1}{2}$. Write down the equation of the tangent to the curve at P. find also the equation of the tangent to the curve whose gradient is $-\frac{1}{2}$, and the coordinates of its point of intersection with the tangent at P.

41. Find the equations of the tangents to the curve $y = x^3 - 6x^2 + 12x + 2$ which are parallel to the line $y = 3x$.

42. Find the coordinates of the points of intersection of the line $x - 3y = 0$ with the curve $y = x\left(1-x^2\right)$. If these points are in order P,O,Q, prove that the tangents to the curve at P and Q are parallel, and that the tangent at O is perpendicular to them.

43. Find the equations of the tangent and the normal to the parabola $x^2 = 4y$ at the point (6,9). Also find the distance between the points where the tangent and the normal meet the y-axis.

44. The curve $y = (x-2)(x-4)(x-3)$ cuts the x-axis at the points P(2,0) , Q(3,0), R(4,0). Prove that the tangents at P and R are parallel. At what point does the normal to the curve at Q cut the y-axis?

45. Find the equation of the tangent at the point P(3,9) to the curve $y = x^3 - 6x^2 + 15x - 9$

If O is the origin and N is the foot of the perpendicular from P to the x-axis, prove that the tangent at P passes through the mid-point of ON. Find the coordinates of another point on the curve, the tangent at which is parallel to the tangent at the point (3,9).

46. A tangent to the parabola $x^2 = 16y$ is perpendicular to the line $x - 2y - 3 = 0$. Find the equation of this tangent and the coordinates of its point of contact.

47. Find the equation of the tangent to $y = x^2$ at the point (1,1) and of the tangent to $y = \frac{1}{6}x^3$ at the point $(2, \frac{4}{3})$. Show that these tangents are parallel, and find the distance between them.

48. The curve C is defined by $y = ax^2 + b$, where a and b are constants. Given that the gradient of the curve at the point (2,-2) is 3, find the values of a and b.

49. Given that the curve with equation $y = Ax^2 + Bx$ has gradient 7 at the point (6,8), find the values of the constants A and B.

50. A curve with equation $y = A\sqrt{x} + \frac{B}{\sqrt{x}}$, for constants A and B, passes through the point (1,6) with gradient -1. Find A and B.

51. Find the equation of the tangent, t, to the curve $y = x^2 + 5x + 2$, which is perpendicular to the line, l, with equation $3y + x = 5$.

END

P425/2
APPLIED
MATHEMATICS
PAPER 2
Nov 2020
3HRS

ST. MARYS' KITENDE
Uganda Advanced Certificate of Education
RESOURCEFUL MOCKEXAMINATIONS 2020
APPLIED MATHEMATICS
PAPER 2
3hours

Instructions to Candidates

- *Answer **ALL** the eight questions in Section A and any **FIVE** from Section B.*
- *All necessary working must be shown clearly*
- *Mathematical tables with a list of formulae and squared papers are provided*
- *In numerical work, take g to be 9.8ms^{-2}*
- *Include the allocation table on your answer sheet*
- *Draw double margins on each of the answer sheets / page to be used.*

SECTION A (40MARKS)

1. Two events A and B are such that $P(A) = 0.7$and $P(\bar{A} \cap \bar{B}) = P(\bar{A} \cup \bar{B}) = 0.2$.
Determine the; i) $P(B)$
ii) $P(\bar{A}/B)$ (5marks)

2. A particle of mass 2kg is acted upon by a force of magnitude 21N in the direction $2i + j + 2k$. Find in vector form the;
i) Force
ii) acceleration hence its magnitude. (5marks)

3. A certain student from S.6 of a certain school recorded the following set of points.

x	-2	-1	0	1	2
y	-5.5	-3.0	1.2	3.4	6.0

Use linear interpolation or extrapolation to estimate;
i) $y$when$x = -0.76$
ii) $x$when $y = 7.8$ (5marks)

4. A discrete random variable X has the probability distribution function given by;

x	5	8	9	11	12
$p(X = x)$	$\boldsymbol{a}$	0.1	$\boldsymbol{a}$	0.4	0.1

Where $\boldsymbol{a}$ is a constant. Find the;
i) Value of a

ii) $E(5x-7)$ (5marks)

5. Car A is travelling at 35ms^{-1} along a straight horizontal road and accelerates uniformly at 0.4ms^{-2}. At the same time, another car B moving at 44ms^{-1}and accelerating uniformly at 0.5ms^{-2}, B is 200m behind A, find the time taken before B overtakes Car A. (5marks)

6. Use the trapezium rule with five ordinates to estimate $f(x) = x + tanx$ from $x = 1$ to $x = 1.4$ correct the value to 3 decimal places. (5marks)

7. The table below shows the time recorded in minutes when Aeroplanes pass through a point of observation at a certain city.

Time	50-	60-	70-	80-	90-	100-	110-120
Frequency	5	3	8	7	10	8	9

Calculate the;

i) median

ii) number of Aeroplanes whose time exceed the median value. (5marks)

8. Forces of magnitude 20N, 12N and 30N act on a particle in the directions due South, east and N40^{0}E respectively. if the fourth force holds the particle in equilibrium; Determine the;

i) magnitude

ii) direction of the forth force. (5marks)

SECTION B (60MARKS)

*Answer any **five** questions in this Section.*

9. The probability density function of a random variable X is given by;

$$f(x) = \begin{cases} \frac{4}{5}x & ;\ 0 < x < 1 \\ \frac{2}{3}(3-x) & ;\ 1 < x < 2 \\ 0 & ;\ otherwise \end{cases}$$

a) Sketch the function f(x) and show that the area =1.

b) Find the mean of x.

c) Determine the cumulative distribution function ($F(x)$). (12marks)

10. a) Two ships A and B are observed from a coast guard station and have the following displacements velocities and times.

Ship	**Displacement**	**Velocity**	**Time(t)**
A	$(i+3j)km$	$(i+2j)kmhr^{-1}$	12:00hours
B	$(i+2j)km$	$(5i+6j)kmhr^{-1}$	13:00hours

Find the time when the two are closest to each other.

b) If at 13:00hours ship A changed its velocity to $\left(\frac{11}{3}i+2j\right)kmhr^{-1}$, show that they collide and find the time and position of collision. (12marks)

11. a) show graphically that the equation $f(x) = 1nx - sinx - 2$ has a root between $x = 3$ and $x = 4$ andestimate the initial approximation (x_0) to 1 decimal place.

b) Using the x_0 above and the Newton Raphson method find the root correct it to 3 decimal places. (12marks)

12. The table below shows the speeds (y) in seconds and the number of errors(x) in the typed scripts of 12 secretaries of a certain institution.

Secretaries	A	B	C	D	E	F	G	H	I	J	K	L
Errors(x)	12	24	20	10	32	30	28	15	18	40	27	35
Speed(y)	130	136	120	120	153	160	155	142	145	172	140	157

a) Construct a scatter diagram, draw the line of best fit and comment hence estimate x when $y = 142$.

b) Giving rank 1 to the fastest secretary and the secretary with the fewest errors calculate the rank correlation co-efficient and comment at 5% level of significance. (12marks)

13. A uniform lamina is in form of a square ABCD of side 2cm. E is a point on AD such that $ED = xcm$, if protion EDC is removed, find the expressions of the location of centre of gravity from AB and from AD, taking AB as the positive y-axis and AD as the positive x-axis. (12marks)

14. a) Given that the numbers x=4.2, y=16.02 and Z=25 are rounded off with corresponding percentage errors 0.5, 0.45 and 0.02 calculate the errors of x, y and Z.

b) Hence find the maximum value, the minimum value, absolute error, relative error and percentage error in $\frac{xy}{z}$. (12marks)

15. The speed of cars passing a certain point on a motorway can be taken to be normally distributed. Observations show that of cars passing the point, 95% are travelling at less than 85kmhr^{-1} and 10% are travelling at less than 55kmh^{-1}. Determine the;

a) mean and standard deviation of the distribution.

b) proportion of cars that travel at more than 70kmhr^{-1} and the percentage it takes. (12marks)

16. A light inextensible string has one end attached to aceiling, the string passes under a smooth moveable pulley of mass 2kg and then over a smooth fixed pulley, the particle of mass 5kg is attached at the free end of the string, the sections of the strings not in contact with the pulleys are vertical, if the system is released from rest and moves in a vertical plane, determine the;

i) accelerations of the 2kg and 5kg masses

ii) tensions of the 2kg and 5kg masses.
iii) distance moved by the system in 1.5 seconds. (12marks)

END

SMASK

S.6 REVISION WORK TERM 1 APPLIED MATHEMATICS P425/2

1. A ship P is moving due west at $12kmh^{-1}$. The velocity of a second ship Q relative to P is $15kmh^{-1}$ in a direction 30^0 west of south.
   Find the velocity of ship Q.

2. (a) Show graphically that the equation $e^{2x} + 4x - 5 = 0$ had only one real root between 0 and 1.
   (b) Use the Newton – Raphson iterative method to find the root of the equation in (a) above giving your answer correct to 2 decimal places.

3. A tennis player hits a ball at a point 0, which is at a height of 2m above the ground and at a horizontal distance 4m from the net, the initial speed being in a direction of 45^0 above the horizontal. If the ball just clears the net which is 1m high,
   (a) Show that the equation of path of the ball is $16y = 16x - 5x^2$.
   (b) Calculate the;
   (i) distance from the net at which the ball strikes the ground.
   (ii) magnitude and direction of the velocity with which the ball strikes the ground.

4. A random variable X has a continuous probability density function (p.d.f) given by

$$f(x) = \begin{cases} c(x+1) & 0 < x < 2 \\ \dfrac{3cx}{2} & 2 \le x < 4 \\ 0 & \text{Elsewhere} \end{cases}$$

Where c is a constant.

a) Sketch the graph of $f(x)$. Hence find the value of c.
b) Determine the cumulative density function (c.d.f) F(x). Hence find the
   (i) Median
   (ii) $P(1.5 < X \le 3)$

5. Given that $M = 8.542, N = 4.6$ rounded off to the given number of decimal places.
a) State the maximum possible error in M and N.

   Determine the interval with in which the value of $M - \frac{M}{N}$ lies.
b) Show that the maximum relative error in the function $\sqrt{MN}$ is given by $\frac{1}{2}\left(\frac{|\Delta M|}{M} + \frac{|\Delta N|}{N}\right)$ where $|\Delta M|$ and $|\Delta N|$ are the respective errors in M and N. Hence calculate the percentage relative error in finding $\sqrt{MN}$.

6. Given that $\tan 30° = 0.577$ and $\tan^{-1}(1.321) = 52.87°$, use linear interpolation or extrapolation to find the value of;
   a) $\tan^{-1}(0.865)$
   b) $\tan 60°$

7. A and B are events such that $P(A) = \frac{8}{15}$, $P(B) = \frac{1}{3}$, and $P(A/B) = \frac{1}{5}$. Find the probability that;
   (i) neither A nor B occurs
   (ii) Event B does not happen if event A has occurred

8. ABCD is a parallelogram. Forces of magnitude 6N, FN and $13\sqrt{2}$ N act along the sides AB, AD and AC respectively in the direction indicated by the order of the letters. Given that angle ADC = 60°, find the value of F and the size of angle BAC.

9. A class contains 18 girls of which 6 are left handed, 12 boys of which 5 are left handed. A group of 5 pupils is selected at random from the class. Find the
   i) probability that at most 2 are girls
   ii) expected number of left handed pupils in the group.

10. A particle Q moves such that its displacement from the origin at any time t is given by ($r = 4t^2\mathbf{i} + 2t\mathbf{j} - 5\mathbf{k}$)m. Find the
    (i) Speed of the particle at t = 3s
    (ii) acceleration of the particle at any time.

11. The price index of an article in 2000 based on 1998 was 130. The price index for the article in 2005 based on 2000 was 80. Calculate the;
    a) price index of the article in 2005 based on 1998
    b) price of the article in 1998 if the price of the article was 45,000 in 2005.

12. Forces of magnitude **5N** and **PN** are acting away from each other at
    an angle of **60°**. Given that their resultant is **7N**, find the:
    (i) value of **P**
    (ii) angle **P** makes with the resultant

13. A discrete random variable X has a cumulative distribution function (c.d.f) given below

x	1	2	3
$F(x)$	β	4β	9β

Find the value of β and the mean of x

14. A particle of weight 20N is on a rough plane. Given that a force of 10N acting away from the plane at 20° to the line of greatest slope is just sufficient to prevent it from sliding down and the frictional force experienced by the particle is 8N, find the angle of inclination of the plane and the reaction between it and the particle.

15. (a) On the same axes, draw a curve $y = xe^x$ and a line $y = x + 1$ to show that the function $xe^x - x - 1 = 0$ has a root in the interval 0 and 1.

(b) Use the Newton Raphson method to find the root of the equation correct to 3 decimal places.

16. The marks scored in an exam are normally distributed with mean 56 and standard deviation 14.2. Find the probability that a candidate picked at random scored

(i) between 62 and 72 marks.

(ii) at least 40 marks.

17. Forces of $(3i+13j)N$, $(2i-j)N$ and $(-i-4j)N$ act at points with position vectors $(i+j)M$, $(3i+2j)M$, and $(-3i+5j)M$. Find the;

(a) magnitude of the resultant force.

(b) the equation of its line of action.

18. $X \sim B(n, p)$ with mean 5 and standard deviation 2. Find the value of n and p.

19. Find the approximate value of $\int_0^1 \frac{1}{x^2+\sin x}dx$ using six-ordinates.

20. (a) Show that the iterative formula based on Newton Raphson formula for finding the root of the equation

$$X = \sqrt[4]{N} \text{ is given by } x_{n+1} = \frac{1}{4}\left(3x_n + \frac{N}{x_n^{3}}\right) \quad n=1, 2, 3\ldots$$

(b) Construct a flow chart that

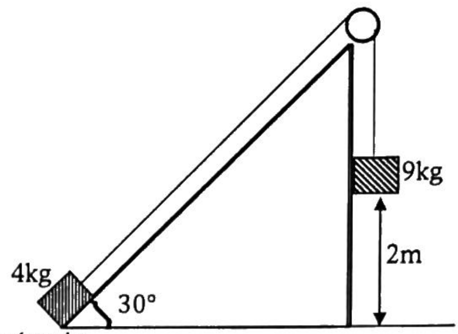
(i) Reads N and the first approximation x_0

(ii) Computes and prints the root to three decimal places or after exactly 4 iterations

Taking $N = 85, x_0 = 3.0$ perform a dry run for the flow chart. Give your root correct to **three** decimal places.

21. Particles of mass 4, 6, 3 and 5kg act at the vertices A, C, B and D respectively of a rectangle ABCD, with $AB = 6\,cm$ and $AD = 4\,cm$. Find the coordinates of the centre of mass of the particles.

22. Two masses of 4 kg and 9 kg are connected by a light inextensible string passing over a smooth pulley at the edge of the smooth inclined plane of inclination 30° as shown below. If the system is released from rest,



Calculate the

(i) acceleration of the system

(ii) tension in the string

(iii) Reaction on the pulley at the edge of the inclined plane.

(iv) Maximum displacement of the 4kg mass up the plane.

23. The table below shows the ages in years of workers on a certain firm.

Age (years)	15–	20–	30–	35–	50–	60–	65–	70–
Frequency density	0.4	0.5	1.6	0.2	0.3	0.8	1.0	0.0

a) Represent the above data on a histogram, hence estimate the modal age.

b) Calculate the

(i) Mean age

(ii) Variance

24. The brakes of a train, which is travelling at 108km^{-1} are applied as the train passes point A. The brakes produce a constant retardation of magnitude $3f$ ms^{-2} until the speed of the train is reduced to 36kmh^{-1}. The train travels at this speed for a distance and is then uniformly accelerated at f ms^{-2} until it again reaches a speed of 108kmh^{-1} as it passes point B. The time taken by the train in travelling from A to B, a distance of 4km, is 4 minutes.

(a) Sketch the speed-time graph for this motion

(b) Calculate;

(i) the value of f

(ii) the distance travelled at 36kmh^{-1}

END

LIRA PALWO SENIOR SECONDARY SCHOOL P.O BOX 34 AGAGO- PADER

END OF TERM III EXAMINATION

Uganda Advanced Certificate of Education

S.5 APPLIED MATHEMATICS

INSTRUCTIONS

- *Answer all the **eight** questions in section A and f**ive** questions from section B.*
- *Any additional question(s) answered will not be marked.*
- *All working must be shown clearly.*
- *Begin each answer on a fresh sheet of paper for section B.*
- *Graph paper is provided.*

SECTION A: (40 MARKS)

1. A body of mass **3kg** and velocity **(8i +7j) ms^{-1}**collides with another body of mass 5kg and velocity **(2i - 5j) ms^{-1}**. If the body collides and coalesce after impact, find the kinetic energy after collision. **(05 Marks)**

2. Events A and B are such that P (A) = $\frac{2}{5}$, P (B'/A) = $\frac{3}{4}$, and P (B/A') = $\frac{1}{3}$. Calculate:

(i) P (A$\cap B$)

(ii) P (B)

(05 Marks)

3. In an examination, scaling is done such that candidate A who had originally scored **35%** gets **50%** and candidate B with **40%** gets **65%**. Determine the original mark for candidate C whose new mark is **80%.**

(05 Marks)

4. A particle, accelerating uniformly with an average velocity of **8 ms^{-1}** for **4 seconds**. If its final velocity is **12 ms^{-1}**, calculate the:

(i) Distance covered

(ii) Acceleration of the particle.

(05 Marks)

5. The table below shows the masses of bolts bought by a carpenter.

Mass(g)	98	99	100	101	102	103	104
N$_{o.}$ of bolts	8	11	14	20	17	6	4

Calculate the:

(a) Median mass

(b) Mean mass of the bolts.

(05 Marks)

6. Use the Newton – Raphson Method to compute $\sqrt{3}$ to 4dps.

(05 Marks)

7. A stone is dropped from the top of a building and at the same time a second stone is projected vertically upwards from the bottom of the building with a speed of **20 ms^{-1}**. If the stones passed each other after **3 seconds**, find the height of the building.

(05 Marks)

8. A set of digits consists of m zeros and n ones. Find the mean of this data and hence show that the standard deviation of the set of digits is

$$\frac{\sqrt{mn}}{m+n}$$

(05 Marks)

SECTION B: (60 MARKS)

9. (a) Use the trapezium rule to estimate the area of $\mathbf{5^{2x}}$ between the x-axis, x =0, and x =1, using **6 ordinates**. Give your answer to 3dps. **(06 Marks)**

(b) Find the exact value of $\int_0^1 5^{2x}\,dx$ **(03 Marks**

(c) determine the percentage error in the two calculations in (a) and (b) above. **(02 Marks)**

(d) How should the error be reduced? **(01 Mark)**

10. X is a discrete random variable such that:

$$P(X=x) = \begin{cases} kx; \ x = 3,4\ 5. \\ k2^x; \ x=1,\ 2. \\ 0; \quad \text{Otherwise} \end{cases}$$

(a) Find the value of the constant k and hence find:

(i) P $(X \geq 2)$

(ii) Variance of X **(09Marks)**

(b) Sketch the graph of P (X=x). **(03Marks)**

11. A pile driver of mass **1200kg** falls freely from a height of **3.6m** and strikes without rebounding a pile of mass **800kg**. The blow drives the pile a distance of **36cm** into the ground. Find the:

(a) Resistance of the ground

(b) Time for which the pile is in motion

[Assume the resistance of the ground to be uniform]

(12 Marks)

12. The table below shows the marks obtained in a Mathematics contest by a group of students in Omot Seed Secondary School.

Marks (%)	0-<20	20-<24	24-<30	30-<32	32-<40	40-<50	50-<60
No. of students	20	24	24	16	16	10	05

(a) Draw a histogram for the data and use it to estimate the modal mark

(06Marks)

(b) Calculate the:

(i) Mean

(ii) Median

(iii) Number of students with marks less than 31%

(06Marks)

13. A particle of mass **10kg** lies on a rough horizontal table and is connected by a light inextensible string passing over a smooth pulley at the edge of the table to a particle of mass 8kg hanging freely. The coefficient of friction between **10kg** mass and the table is **0.25**. If the system is released from rest with **10kg** mass at the edge of the table, **1.5m** from the pulley, find the:

(i) Acceleration of the system

(ii) Reaction on the pulley

(iii) Speed of the 10kg mass as it reaches the pulley.

(12Marks)

14. The table below shows the expenditure of restaurant for the year 2020 and 2021.

Item	**Prices (shs)**		**Weight**
	2020	2021	
Milk (per liter)	1000	1300	0.5
Eggs (per tray)	6500	8300	1.0
Sugar (per kg)	3000	3800	2.0
Blue band	7000	9000	1.0

Taking 2020 as the base year, calculate for 2021 the:

(a) (i) price relative for each item **(04Marks)**

(ii) Simple aggregate price index **(03 Marks)**

(iii) Weighted aggregate price index and comment on your result. **(03 Marks)**

(b) In 2021, the restaurant spent shs. 4500 on buying these items. Using the index obtained in (a) (iii) above, find how much the restaurant could have spent in 2020. **(02 Marks)**

15. A car travels along a straight horizontal road, passing two garages, A and B. The car passes A at **u ms^{-1}** and maintains this speed for **60 seconds**, during which time it travels **900m**. Approaching a junction, the car then slows at a uniform rate of **a ms^{-2}** over the next **125m** to reach a speed of **10ms^{-1},** at which instant, with the road clear, the car accelerates at a uniform rate of **0.75ms^{-2}.** This acceleration is maintained for **20seconds** by which the time the car has reached a speed of **v ms^{-1},** which is then maintained. The car passes B **45seconds** later after its speed reaches v ms^{-1}.

(i) Calculate the value of **u, a,** and **v.**

(ii) Sketch a velocity-time graph for the motion of the car between A and B.

Find the distance between garages A and B and the time taken by the car to travel this distance.

(12 Marks)

16. (a) If a is the first approximation to the root of the equation $\mathbf{X^5 - b = 0}$, show that the second approximation is given by $\frac{4a+\frac{b}{a4}}{5}$ **(04 Marks)**

(b) Show that the positive real root of the equation $\mathbf{X^5 - 17 = 0}$ lies between 1.5 and 1.8. Hence use the formula in (a) above to determine the root to three decimal places.

(08 Marks)

THE END

WISH YOU A MERRY CHRISTMASS AND PROSPEROUS NEW YEAR 2023

OUR LADY OF AFRICA S.S NAMILYANGO (OLAN)

A LEVEL APPLIED MATHEMATICS P425/2 SEMINAR QUESTIONS 2022

PROBABILITY

1. (a). Two events A and B are such that $P(A) = \frac{8}{15}$, $P(B) = \frac{1}{3}$, and $P(A/B) = \frac{1}{5}$. Calculate the probabilities that;

(i). Both occur

(ii).Only one of the two events occurs

(iii).Neither of the events occurs

(b). A box contains 4 pink counters, 3 green counters and 3 yellow counters. Three counters are drawn at random one after the other without replacement

(i). Find the probability that the third counter drawn is green and the first two are of the same colour.

(ii).Find the expected number of pink counters drawn.

2. (a).In an interview, 30% of the students failed and 10% of the students achieved the distinctions. Last year, the pass mark was 84 and the minimum distinction mark was 154. If the marks of the students were normally distributed, find the probability that a student chosen at random scored more than 155 marks.

(b). If 20% of loan applicants received by the bank are rejected, find the percentage that among 450 loan applications, only 90 will be rejected.

3. (a). Two biased tetrahedrons have each of their faces numbered 1 to 4. The chance of getting any one face showing uppermost is inversely proportional to the number on it. If the two tetrahedrons are thrown and the number on the upper most face noted, determine the probability that the faces show the same number.

(b). If it is a fine day, the probability that Sir Fred goes to play football is $\frac{9}{10}$ and the probability that Bob goes is ¾. If it is not fine, Sir Fred's probability is ½ and Bob's is ¼ . Their decisions are independent. In general it is known that it is twice as likely to be fine as not fine.

(i). Determine the probability that both go to play

(ii). If they both go to play, what is the probability that it is a fine day.

4. (a). X is a discrete random variable which takes all integers from 1 to 40 such that $P(X = x) = kx;\ x = 1,2,3, \dots ,40$.

(i)Find the value of the constant k.
(ii).Compute the standard deviation of X.
(iii). Find $P(x < 35/x > 20)$.
(b). X is a random variable such that;

$$f(x) = \begin{cases} \beta(1-2x); & -1 \le x \le 0 \\ \beta(1+2x); & 0 \le x \le 2 \\ 0 & ; elsewhere. \end{cases}$$

(i) Sketch the pdf, $f(x)$
(ii) Determine the value of the constant, β
(iii) Find the mean of x
(iv) Find the 60^{th} $Percentile$

STATISTICS

5. (a). For a particular set of observations $\sum f = 20$, $\sum fx^2 = 16143$ and $\sum fx = 563$. Find the value of the;

(i). Mean

(ii). Standard deviation

(b). A set of digits consists of m zeros and n ones.

(i). Find the mean of this set of data.

(ii). Hence show that the standard deviation of the set of digits is $\frac{\sqrt{mn}}{m+n}$

(c). A physics student measured the time required in seconds for a trolley to run down slopes of varying gradients and obtained the following results.

32.5, 34.5, 33.5, 29.3, 30.9, 31.8. Calculate the mean time and standard deviation

6. (a). The table below shows the distribution of weights of a group of animals.

Mass(kg)	Frequency
21-25	10
26-30	20
31-35	15
36-40	10
41-50	30

51-65	45
66-75	5

(i) Construct a histogram for the above data and use it to estimate the mode.

(ii) Calculate the median for the above data.

(b)The distribution below shows the weights of babies in Gombe hospital 3, 5, 3, 9, 6, 8, 20, 19, 24, 14, 12. Find the;

(i). Upper quartile (ii). Lower quartile (iii). Median (iv). Variance

(v). Standard deviation.

7. Eight candidates seeking admission to a university course sat for a written and oral test. The scores were as shown in the table below.

Written X	55	54	35	62	87	53	71	50
Oral Y	57	60	47	65	83	56	74	63

(i). Draw a scatter diagram for this data

(ii). Draw a line of best fit on your scatter diagram

(iii). Use the line of best fit to estimate the value of Y when X=70.

(iv). Calculate the rank correlation coefficient. Comment on your result.

8. The table below shows the expenditure of restaurant for the year 2014 and 2016.

Item	Prices(shs)		weight
	2014	2016	
Milk (per litre)	1000	1300	0.5
Eggs (per tray)	6500	8300	1
Sugar (per kg)	3000	3800	2
Blue band	7000	9000	1

Taking 2014 as the base year, calculate for 2016 the;

i. Price relative for each item

ii. Simple aggregate price index
iii. Weighted aggregate price index and comment on your result
iv. In 2016, the restaurant spent shs. 4,500 on buying these items. Using the index obtained in (iii), find how much the restaurant could have spent in 2014.

NUMERICAL ANALYSIS

9(a). Use the trapezium rule to estimate the area of 5^{2x} between the $x - axis, x = 0, and\ x = 1$, using 5 sub-intervals. Give your answer correct to 3dps.

(b). Find the exact value of $\int_0^1 5^{2x} dx$

(c). Determine the percentage error in the two calculations in (a) and (b) above.

(d). How should the error be reduced.

10(a). Show graphically that the $x^3 + 5x^2 - 3x - 4 = 0$ has roots between 0 and -1.

(b). Use the Newton Raphson method to calculate the root of the equation in (a) correct to 2dps.

11(a). Given that $y = sec(45^0 \pm 10\%)$, find the limit within which the exact value of y lies

(b). Construct a flow chart that computes and prints the average of the squares of the first six counting numbers. Perform a dry run for your flow chart.

12. If the numbers x and y are approximations with errors Δx and Δy respectively;

(a). Show that the maximum absolute error in the approximations of x^2y is given by $|2xy\Delta x| + |x^2y|$. Hence find the limits within which the true value of x^2y lies given that $x = 2.8 \pm 0.016\ and\ y = 1.44 \pm 0.008$.

(b) Given two numbers $x = 3.815\ and\ y = 2.43$, find the absolute error in the quotient $\frac{y}{x}$, truncate to 3dps. Hence, find the least and greatest value of $\frac{y}{x}$, correct to two decimal places.

MECHANICS

13. (a) At 8:00am , a bus initially packed at a stage A, starts moving along a straight road with acceleration $a = (4t)kmh^{-2}$, which acceleration continues until $t = 5hours$, where upon it cease and the bus uniformly retards at $20kmh^{-2}$ to stage B, Determine the;
(i) Time when the bus reaches stage B.
(ii) Distance between A and B

(iii) Sketch a velocity- time graph to represent the above journey of the bus.

14. Two bodies P and Q are simultaneously projected from a point O with the same speed but at different angles of elevation, and they both pass through a point c which is at a horizontal distance $2h$ from O and at a height h above the level of O. The body P is projected at an angle $tan^{-1}(2) above\ the\ horizontal$. Show that;

(a) The speed of projection is $\sqrt{\frac{10gh}{3}}$

(b) Q is projected at an angle $tan^{-1}\left(\frac{4}{3}\right)$ to the horizontal

(c) The time interval between the arrivals of the two bodies at C is $(3-\sqrt{5})\sqrt{\frac{2h}{3g}}$

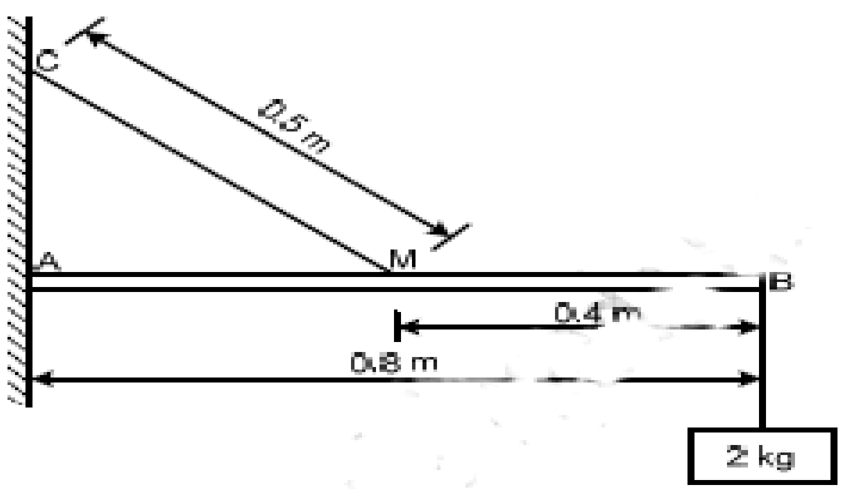
15. At 10:00am, ship A moving at $20kmh^{-1}$ due east is 10km south east of another ship B. If B is moving at $14kmh^{-1}$ in the direction S30^0W and the ships maintain their velocities, find the;

(a) time when the ships are closest together,

(b) shortest distance between the ships,

(c) bearing of A from B at that time.

16. The figure below shows a uniform beam of length 0.8m and mass 1kg. the beam is hinged at A and has a load of mass 2kg attached at B.



The beam is held in a horizontal position by a light inextensible string of length 0.5m. The string joins the midpoint M of the beam to a point C vertically above A. Find the

(i) Tension in the string

(ii) Magnitude and direction of the force exerted by the hinge

17. The resultant of the forces $F_1 = \binom{3}{a-c}, F_2 = \binom{2a+3c}{5}, F_3 = \binom{4}{6}$ acting on a particle is $\binom{10}{12}$. *Find the*;

(i) Values of a and c

(ii) Magnitude of force F_2

(b) Five forces of magnitudes 3N, 4N, 4N, 3N, and 5N, act along the lines AB, BC, CD, DA, and AC respectively, of a square ABCD of side 1m. the direction of the forces is given by the order of the letters. Taking AB and AD as reference axes, find the magnitude and direction of the resultant force.

WISHING YOU GREAT SUCCESS IN YOUR UNEB EXAMINATIONS

END

P425/2
APPLIED MATHEMATICS
Paper 2
Mar/Apr. 2022
3 HOURS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

APPLIED MATHEMATICS

(PRINCIPAL SUBJECT)

Paper 2

TIME: 3 HOURS

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **Eight** *questions in Section* **A** *and* **Five** *questions from Section* **B**.

Any additional question(s) answered will **not** *be marked.*

All *necessary working* **must** *be clearly shown.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take g *to be* $9.8\ ms^{-2}$.

SECTION A: (40 MARKS)

Answer **all** *the questions in this section*

1. Two events A and B are such that P(A)$= 0.6$, P(AnB)$= 0.4$ and $P(\bar{A}n\bar{B}) = 0.2$, find;
   (i) $P(\bar{B})$
   (ii) P(A *or* B but not both A and B)

2. P, Q and R are points on a straight road such that $PQ = 2$km and $QR = 2km$. A cyclist moving with uniform acceleration passes P and then notices that it takes him 100s and 150s to travel between (P and Q) and (Q and R) respectively. Find
   (i) uniform acceleration
   (ii) initial velocity.

3. The data below represents the length of leaves in cm, $4.4, 6.2, 9.4, 12.6, 10.0, 8.8, 3.8$ and 13.6. Find the;
   (i) Mean
   (ii) Standard deviation

4. Show that the equation $f(x) = x^3 - 3x - 12$, has a root between $x = 2$ and $x = 3$. Hence using linear interpolation once, estimate the root to two decimal places.

5. Forces of magnitude 5N, $2\sqrt{3}N$, 16N and 4N act on a body in the direction of $\mathbf{0}°$, $6\mathbf{0}°$, $12\mathbf{0}°$ and $27\mathbf{0}°$ respectively. Determine the;
   (a) Resultant force and magnitude
   (b) Direction of the resultant force

6. The table below shows the expenditure of a certain family for the January and March 2022

Items	Expenditure (Shs)		Weight
	January	March	
Food	300,000	325,000	5
Accommodation	260,000	362,000	3
Electricity	150,000	160,000	1
Miscellaneous	620,000	725,000	2

   (a) Calculate the weighted expenditure index for the month of March based on January 2022
   (b) If the expenditure in January was 1,350,000/=, use the index above to estimate the expenditure in March.

7. A particle of weight 6N is placed on a rough plane inclined at 45° to the horizontal, if the coefficient of friction between the particle and the plane is $\frac{1}{2}$. Find the magnitude of the least horizontal force required to maintain the particle in equilibrium

8. The table below is an extract from the table of $cosx$.

	0^1	10^1	20^1	30^1	40^1	50^1
80^0	0.1736	0.1708	0.1679	0.1650	0.1622	0.1593

Use linear interpolation to determine

(i) $cos80°36'$ (ii)$cos^{-1}(0.1685)$.

SECTION B: (60 MARKS)

*Answer any **five** questions in this section. All questions carry **equal** marks*

9. The numbers a and b have errors e_1 and e_2 respectively.

(a) Show that the maximum relative error made in the approximation of $\frac{a}{\sqrt{b}}$ is

$$\left|\frac{e_1}{a}\right| + \frac{1}{2}\left|\frac{e_2}{b}\right|$$

(b) Given that $a = 42.3, b = 27.31$ and $c = 12.333$, are rounded off to the given decimal places, find the maximum value and minimum value hence percentage error in the expression $\frac{a}{b+c}$

10. Two particles P and Q move in a straight line. Q being 18m in front of P, Q starts from rest with acceleration of $3ms^{-2}$ and P starts in pursuit with a velocity of $10ms^{-1}$ and an acceleration of $2ms^{-2}$. Prove that;

(i) P will overtake and pass Q after an interval of 2 seconds

(ii) Q will in turn over take P after a further interval of 16 seconds

11. (a) A bag contains 5 blue and 4 red balls, 3 balls are drawn from the bag one at a time without replacement. Determine the probability

(i) All are blue (ii) 2 are red and 1 is blue

(b) Daniel's probability of passing Physics, Economics and Mathematics are 0.6, 0.75 and 0.8 respectively, find the probability that he;

(i) Passes at least two subjects

(ii) Failed economics given that he passed at least two subjects

12. The data below represents the body mass (x) and the heart mass (y) of fourteen 10 month old male mice

Body mass (x)	27	30	37	38	32	36	32	32	38	42	36	44	33	38
Heart mass (y)	118	136	156	150	140	155	157	114	144	159	149	170	131	160

(a) Construct a scatter diagram, draw the line of best fit and comment on it

(b) Estimate the heat mass (y), if the body mass (x) is 35

(c) Calculate the rank correlation and comment at 1% level of significance

13. (a) Use Newton Raphson formulae, derive the simplest iterative formula to determine the cube root of a number A is given by;

$$\boldsymbol{x_{n+1}} = \frac{1}{3}\left(2\boldsymbol{x_n} + \frac{\boldsymbol{A}}{x_n{}^2}\right); n = 0, 1,2 \ldots$$

(b) Using $\boldsymbol{x_0} = \boldsymbol{4}$ and $A = 68$ using Newton Raphson method, find the root and Correct it to 3 decimal places

14. A particle of mass 2m rests on a rough plane inclined to the horizontal at an angle of $tan^{-1}(3\mu)$ where μ is the coefficient of friction between the particle and the plane. The particle is acted upon by a force of P Newton's.

(a) Given that the force acts along the line of greatest slope and that the particle is on the point of slipping up, show that the maximum force possible to maintain the particle in equilibrium is $P_{max} = \frac{8\mu mg}{\sqrt{1+9\mu^2}}$

(b) Given that the force acts horizontally in a vertical plane through a line of greatest slope and that the particle is on the point of sliding down the plane, show that the minimum force required to maintain the particle in equilibrium is $P_{min} = \frac{4\mu mg}{1+3\mu^2}$

15. A random variable X takes integer values only and has probability density function given as;

$$P(X{=}x) = \begin{cases} kx^2; & x = 1, 2, 3 \\ k(7-x)^2; & x = 4,5,6 \\ 0; & otherwise \end{cases}$$

Where k is a constant, determine the;

(i) Value of k

(ii) E(X) and Var (X)

(iii) Median and mode

(iv) $P\left(x \le 3 / x \le 5\right)$

16. ABCDEF is a regular hexagon of side 4m, Forces of magnitude 2N, 6N, 5N, 8N, and 3N act along the sides AB, BC, CD, ED and EF respectively. Taking AB and AE as the reference positive x and y axes respectively, determine the;

(a) Magnitude of the resultant force and its direction

(b) Line of action of resultant force by taking moments about A and point where it cuts AB

P425/1
PURE MATHEMATICS
Paper 1
July/Aug. 2020
3 hours

ASK INTERGRATED TEACHERS' MOCK EXAMINATIONS BURREAU

AITEL JOINT MOCK EXAMINATIONS

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES

Answer **all** *the* **eight** *questions in section* **A** *and any* **five** *questions from section* **B**

Any additional question(s) answered will **not** *be marked*

Show **all** *the necessary workings clearly*

Begin each question on a fresh page of paper

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used

 Turn Over

SECTION A (40 MARKS)

Attempt **all** *questions in this section*

1. Given that $a=\log_5 35$ and $b=\log_9 35$, show that $\log_5 21=\frac{1}{2b}(2ab-2b+a)$.

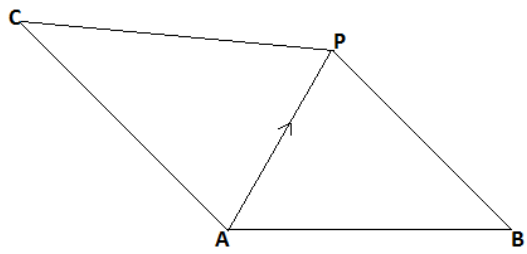
(05 marks)

2. Solve the inequality: $|x-2|>3|2x+1|$. (05 marks)

3. Prove that: $\tan^{-1}\frac{1}{2}-\cos ec^{-1}\frac{\sqrt{5}}{2}=\cos^{-1}\frac{4}{5}$. (05 marks)

4. Expand $\frac{1}{\sqrt{1+x}}$ up to the term in x^2 and by letting $x=\frac{1}{4}$, show that $\sqrt{5}\approx\frac{256}{115}$. (05 marks)

5.



$A,\ B,\ C$ and P are four points such that $3\overrightarrow{\mathbf{AP}}=2\overrightarrow{\mathbf{AB}}+\overrightarrow{\mathbf{AC}}$, show that $B,\ P$ and C are collinear and that P is the point of trisection of the line BC

. (05 marks)

6. Given that $y=\frac{e^x-e^{-x}}{e^x+e^{-x}}$, show that $\frac{d^2y}{dx^2}+2y\frac{dy}{dx}=0$. (05 marks)

7. Find the volume of the solid generated by rotating the area bounded by the curve $y=\cos\frac{1}{2}x$ from $x=0$ to $x=\pi$ about the $x-$ axis. (05 marks)

8. Solve the d.e given $\cos x\frac{dy}{dx}-2y\sin x=1$. (05 marks)

SECTION B (60 MARKS)

Attempt any **five** *questions in this section*

9 (a) Evaluate the coefficient of x in the expansion of $\left(x+\frac{2}{x^2}\right)^{10}$. (05 marks)

(b) Prove by Mathematical induction that: $\sum_{r=2}^{n}\frac{1}{r^2-1}=\frac{3}{4}-\frac{2n+1}{2n(n+1)}$. (07 marks)

10 (a) Prove the identity: $\cos^6 x+\sin^6 x=1-\frac{3}{4}\sin^2 2x$. (06 marks)

(b) Solve the equation: $4\sin^2 x+8\cot^2 x=5\operatorname{cosec}^2 x$ for $0\le x\le 2\pi$. (06 marks)

11. Sketch the curve $y=\frac{4+3x-x^2}{x-8}$, clearly find the nature of the turning points and state their asymptotes. (12 marks)

12 (a) The points $P\left(5p,\ \frac{5}{p}\right)$ and $Q\left(5q,\ \frac{5}{q}\right)$ lie on the rectangular hyperbola $xy=25$. Find the equation of the tangent at P and hence deduce the equation of the tangent at Q. (05 marks)

(b) The tangents at P and Q meet at point N, show that the coordinates of N are $\left(\frac{10pq}{p+q},\ \frac{10}{p+q}\right)$, hence find the locus of N given that $pq=-1$.

(07 marks)

13 (a) Given that $z(5+5i)=a(1+3i)+b(2-i)$ where a and b are real numbers and that $\arg z=\frac{\pi}{2}$ and $|z|=7$, find the values of a and b. (06 marks)

(b) Describe the locus of the complex number z when it moves in the argand diagram such that $\arg\left(\frac{z-3}{z-2i}\right)=\frac{\pi}{4}$. (06 marks)

14 (a) Evaluate: $\int_0^{\pi/3} x\sin 3x\,dx$ (06 marks)

Turn Over

(b) Prove that: $\int_{\pi}^{4\pi/3} \operatorname{cosec}\frac{1}{2}x \, dx = In3$ (06 marks)

15 (a) Find the point of intersection between the plane $\mathbf{r}\cdot(2\mathbf{i}-\mathbf{j}+\mathbf{k})=\mathbf{4}$ and the line passing through the point $(3,\ 1,\ 2)$ and is perpendicular to this plane.

(05 marks)

(b) Find the perpendicular distance of the point $(4,\ -3,\ 10)$ to the line

$$\frac{x-1}{3}=2-y=\frac{z-3}{2}.$$ (07 marks)

16. A liquid is being heated in an oven maintained at a constant temperature of $180^{o}C$. It is assumed that the rate of increase in the temperature of the liquid is proportional to $(180-\theta)$, where $\theta^{o}C$ is the temperature of the liquid at time t minutes. If the temperature of the liquid rises from $0^{o}C$ to $120^{o}C$ in 5 minutes, find the temperature of the liquid after a further 5 minutes. (12marks)

END

P425/1
PURE MATHEMATICS
Paper 1
July/Aug. 2022
3 hours

AITEL JOINT MOCK EXAMINATIONS

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 Hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in the section* **A** *and*

Answer any **five** *questions in section* **B**

 Turn Over

SECTION A (40 MARKS)

1. The roots of a quadratic equation $x^2 + (7+p)x + p = 0$ are $\propto$ and β.
   Given that $\propto$ and β differ by 5, find the possible values of p. (5 marks)

2. Evaluate $\int_2^3 \frac{2}{x^2 - 4x + 13} dx$ (5 marks)

3. Show that the parametric equations $x = 5 + \frac{\sqrt{3}}{2}\cos\theta \;\; y = -3 + \frac{\sqrt{3}}{2}\sin\theta$ represent a circle. Find the radius and centre of the circle. (5 marks)

4. Solve the equation: $3 + 2\cos^2\theta + \tan\theta = 4\cos^2\theta$, for $0^0 \leq \theta \leq 360^0$ (5 marks)

5. How many team of 6 players can be formed from a group of 7 boys and 5 girls if
   (i) Each team should have at least 3 boys and a girl
   (ii) Each team contains at most 3 girls (5 marks)

6. Use raw Echelon reduction to solve the following Equations simultaneously.
   (i) $3x = z - 2y$
   (ii) $3y = x + 2z + 1$
   (iii) $3z = 2x - 2y + 3$ (5 marks)

7. Differentiate $y = x\ln x$ from first principles . (5 marks)

8. Find the equation of the normal to the curve $x^2\tan x - xy - 2y^2 = -2$ at the point (0,1) (5 marks)

SECTION B (60 MARKS)

Answer any **five** *questions*

9. (a) Show that the lines $r = 5 + 3 - 5k + u(+2j - 3k)$ and $\frac{x-7}{3} = \frac{y+1}{-2} = \frac{z+4}{-2}$ intersect and hence find the coordinates of the point of intersection. (7 marks)

   (b) A plane is at a distance of $\sqrt{11}$ units from the origin. If theline passing through the points A(4,-9,3) and B(6,-7,9) is perpendicular to the plane, find the cartesian equation of the plane. (5 marks)

10. (a) Show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ hence solve the equation $8x^3 - 6x - 1 = 0$
    (7 marks)

    (b) By expressing 5cosx + 8sinx in the form $Rcos(x + \beta)$, where R is a constant and β is an acute angle, solve $5cosx + 8sinx = 7$
    for $0^0 \leq x \leq 360^0$ (5 marks)

11. Evaluate $\int_2^3 \frac{2x+1}{(x-2)(x+1)^2}dx$. Give your answer correct to 3 decimal places.

(12 marks)

12. (a) Prove by induction that $4^{(n+3)} - 3n - 10$ is divisible by 3 for all positive integral values of *n*. (6 marks)

(b) Use De Moivres theorem to prove that the complex number $(\sqrt{3}+i)^n + (\sqrt{3}-i)^n$ is always real and hence find the value of the expression when n = 6. (6 marks)

13. (a) Use the binomial theorem to show that

$$(\sqrt{1+2x} + \sqrt{1-4x})^2 = (2 - x - \frac{5}{2}x^2 + ...)^2$$ (7 marks)

(b) Taking $x = \frac{1}{16}$ use the equation in (a) above to estimate $\sqrt{6}$ to 2 decimal places.

(5 marks)

14. (a) Solve for x in; $9^x - 3^{(x+1)} = 10$ (5 marks)

(b) Solve the following pair of simultaneous equations.

$$\log_2 x^2 + \log_2 y^3 = 1$$
$$\log_2 x - \log_2 y^2 = 4$$

(7 marks)

15. (i) If the curve $y = \frac{x^2 - 4x + 4}{x+1}$ Show that the curve is restricted, state the region and hence investigate the nature of the turning points.

(ii) Determine the equations of asymptotes to the curve

(iii) Sketch the curve

(12 marks)

16. (a) Solve the differential equation.

$y\cos^2 x \frac{dy}{dx} = \tan x + 2$, given that y = 0 when $x = \frac{\pi}{4}$ (5 marks)

(b) A spherical bubble evaporates at a rate proportional to its surface area. If half of it evaporates in 2 hours, when will the bubble disappear? (7 marks)

END

P425/2
APPLIED
MATHEMATICS
Paper 2
July/Aug. 2022
3 hours

ASK INTERGRATED TEACHERS' MOCK EXAMINATIONS BURREAU

AITEL JOINT MOCK EXAMINATIONS

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 Hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in the section* **A** *and*

Answer any **five** *questions in section* **B**

In numerical work, take g= 9.8ms^{-2}

 Turn Over

SECTION A (40 MARKS)

1. Forces 7N and 9N act on a particle at an angle of 60^0 between them.
Find the magnitude and the direction of the resultant. (5 marks)

2. Show that the equation $3x^2 + x - 5 = 0$ has a real root between $x = 1$ and $x = 2$ hence determine the root to two decimal places using linear interpolation. (5 marks)

3. A game consists of throwing tennis balls into a bucket from a given distance, the probability that Joan will get the tennis ball in the bucket is 0.4. A turn consists of three attempts, construct a probability distribution for X, the number of tennis balls that lands into the bucket.
(5 marks)

4. An object is projected from the ground making an angle $\tan^{-1}\left(\frac{3}{4}\right)$ with the horizontal. The particle passes through a paint 20m horizontally and 10m vertically form the point of protection. Find the speed of projection.
(5 marks)

5. A bullet of mass 20g is fired into a block of wood of mass 40g lying on smooth horizontal surface. If the bullet and the wood move together with speed $20ms^{-1}$. Calculate the loss in kinetic energy. (5 marks)

6. Obtain the interval of values with in which the exact value of the expression $\frac{15.36 + 27.1 - 1.672}{2.36 \times 1.043}$ lies to 3.d.ps. (5 marks)

7. Event A and B are such that P(A) $= \frac{4}{7}$, $P(A \cap B^1) = \frac{1}{3}$ and $P(A|B) = \frac{5}{14}$.
Find;
(i) $P(A \cup B)$
(ii) $P(A^1 \cap B^1)$ (5 marks)

8. The following shows the marks obtained by eight students in Mathematics and physics examinations.

Maths	55	65	70	75	75	80	85	85
Physics	50	55	58	55	65	58	61	65

Calculate the rank correlation coefficient and comment on the significance of your result at 5% level of significance. (5 marks)

SECTION B (60 MARKS)

9. A body of mass Mkg lies on a rough plane inclined at θ^0 to the horizontal when a force of $\frac{Mg}{2}$ newtons parallel to and up the plane is applied to the body, it is just about to move up the plane. When a force of $\frac{Mg}{4}$ newtons parallel to and down the plane is applied to the body, it is just about to move down the plane. Calculate to two decimal places the value of;

(i) θ (8 marks)

(ii) The coefficient of friction between the body and the plane. (4 marks)

10. The age of farmers in Nkoowe town Wakiso district are as shown in the table below.

Age (years)	**Number of farmers**
25 - < 29	6
29 - < 35	12
35 - < 40	27
40 - < 50	30
50 - <55	18
55 - < 60	14
60 - < 70	9
70 - < 75	4
75 - < 80	5

(a) Determine the variance and the modal age. (6 marks)

(b) Construct a cumulative frequency curve and use to estimate the probability that a farmer randomly selected from Nkoowe town has age above47 years. (6 marks)

11. In a box containing different balls, the probability that the ball is red is 0.3. If a sample of 400 balls is selected from the box. Find the probability that;

(i) Less than 120 balls are red

(ii) Between 120 and 150 inclusive are red

(iii) More than 160 balls are red

12. (a) Use graphical method to locate the positive root of the equation $\sin x - \ln x = 0$ to 1 decimal place. (6 marks)

(b) Use the Newton- Raphson method to find the root of the equation to 3 decimal places.

13. (a) Use trapezium rule with 6 ordinates to estimate the value of $\int_1^3 x^2 \ln x dx$ correct to 3 decimal places. (6 marks)

(b) Find the error made in the above estimate and suggest ways of reducing the error.

(6 marks)

14. A continuous random variable has a p.d.f

$$f(x) = \begin{cases} cx, & 0 \le x \le 2 \\ c(4-x), & 2 \le x \le 4 \\ 0, & elsewhere \end{cases}$$

Determine,

(i) The value of c. (4 marks)

(ii) The mean of X (4 marks)

(iii) Upper quartile Q_3 (4marks)

15. A particle moving with an acceleration $a = 4^{e-3t}\mathbf{i} + 12\sin t\mathbf{j} - 7\cos t\mathbf{k}$ is located at a point (5,-6,2) and has velocity $V = 11\mathbf{i} - 8\mathbf{j} + 3\mathbf{k}$ at t = 0. Find the

(i) Magnitude of acceleration when t = 0

(ii) Velocity at any time, *t*

(iii) Displacement at any time, *t* (12 marks)

16. A uniform ladder of length 12m and mass 10kg, one end A resting on a rough horizontal surface and end B resting on a rough vertical wall. Coefficient of friction between the ladder and the wall is $\frac{1}{4}$ and coefficient of friction between the ladder and the horizontal surface is $\frac{1}{2}$. Find how far a man who is ten times the mass of the ladder can climb up the ladder before it slips. The angle between the ladder and the horizontal is 60^0

(12 marks)

END

P425/1
PURE MATHEMATICS
Paper 1
Sept. 2022
3 hours

VECTORS TEST

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

*Answer **all** the **eight** questions in section **A** and only **five** in section **B**.*

*Any additional question(s) answered will **not** be marked.*

*Each question in section **A** carries **5** marks while each question in section **B** carries **12** marks.*

*All working **must** be shown clearly.*

Begin each answer on a fresh sheet of paper.

Silent , non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

TURN OVER

SECTION A: (40 MARKS)

*Attempt **all** questions in this section.*

1. Show that the points A(1,2,3), B(3,8,1) and C(7,20,-3) are collinear.

   (05 marks)

2. OAB is a triangle. X and Y are the midpoints of **OA** and **AB** respectively. If $\boldsymbol{a}$ and $\boldsymbol{b}$ are position vectors of points A and B respectively. Find in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ the position vector of Z, the point of intersection of **XB** and **OY**.

   (05 marks)

3. Find the angle between the y-axis and the line $\boldsymbol{r} = \boldsymbol{i} + 2\boldsymbol{k} + \lambda(4\boldsymbol{i} + 6\boldsymbol{j} - 2\boldsymbol{k})$. State the direction ratios of the given line. *(05 marks)*

4. Show that the lines $\boldsymbol{r_1} = \begin{pmatrix} 17 \\ 2 \\ -6 \end{pmatrix} + \alpha \begin{pmatrix} 2 \\ -3 \\ 9 \end{pmatrix}$ and $\boldsymbol{r_2} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} + \beta \begin{pmatrix} 6 \\ 7 \\ -1 \end{pmatrix}$ are skew. *(05 marks)*

5. Find the equation of plane in scalar product form that contains the points P(1,1,1), Q(1,-2,-1) and R(3,-1,2). *(05 marks)*

6. Find the foot of the perpendicular from the point A(25,5,7) to the plane with equation $12x + 4y + 3z = 3$. *(05 marks)*

7. Show that the points M(2,21) and N(1,-2,1) lie on the opposite sides of the plane with equation $2x - 3y + z = 1$. *(05 marks)*

8. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-1}{1}, z = 1$. *(05 marks)*

SECTION B: (60 MARKS)

*Attempt only **five** questions from this section.*

9. (a) Show that the points A,B and C with position vectors $3\boldsymbol{i} - 4\boldsymbol{j}, 2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$ and $7\boldsymbol{i} + 4\boldsymbol{j} - \boldsymbol{k}$ are vertices of a triangle. Hence find the area of the triangle. *(07 marks)*

   (b) PQRS is a quadrilateral with P(2,2), Q(5,-1),R(6,-2) and R(3,1). Identify the type of quadrilateral using vector methods. *(05 marks)*

10. (a) Find the coordinates of the point of intersection of the plane $2x - y + 4z = 32$ and the line $\frac{1-x}{-3} = \frac{y+2}{2} = \frac{z-1}{5}$. Obtain the angle between the line and the plane. *(08 marks)*

    (b) Find whether the points A(2,-2,5) and B(1,2-1) lie on the plane in (a) above. *(04 marks)*

11. The equations of the line L and the plane π are $\frac{x-2}{1} = \frac{2-y}{-2} = \frac{z-3}{3}$ and $2x + y + 4z = 13$. P is a point on the line where $x = 3$. N is the foot of the perpendicular from point P to the plane. Find

    (a) The coordinates of the point N. *(08 marks)*

    (b) Equation of the plane through (1,2,-1) perpendicular to line **NP**. *(04 marks)*

12. The vector equations of 2 lines are $\boldsymbol{r_1} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ and $\boldsymbol{r_2} = \begin{pmatrix} 2 \\ 2 \\ t \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ where t is a constant. If the two lines intersect at C, find

    (a) The value of t and the coordinates of C *(07 marks)*

    (b) Find the cartesian equation of the plane containing the two lines. *(05 marks)*

13. (a) The points A and B has position vectors a and b respectively with respect to the origin O. Point C divides line **AB** externally in the ratio $\lambda : \mu$. Show that $\boldsymbol{OC} = \frac{\lambda \boldsymbol{b} - \mu \boldsymbol{a}}{\lambda - \mu}$. *(04 marks)*

(b) The coordinates of points A and B are (-3, -1, -2) and (5,3,6) respectively. If point P divides **AB** externally in the ratio 2:3 and point Q divides **AB** externally in the ratio 5:1, find the coordinates of R the midpoint of **PQ**. *(05 marks)*

(c) Find the equation of line through point R in (b) above and perpendicular to the plane $2x - 3y + z = 5$. *(03 marks)*

14. A triangle OPR is such that $\boldsymbol{OR} = \boldsymbol{r}$ and $\boldsymbol{OP} = \boldsymbol{p}$**.** S and Q are points produced on **OR** and **OP** such that $\boldsymbol{OS} = 2\boldsymbol{r}$ and $2\boldsymbol{OQ} = 3\boldsymbol{p}$ respectively. K is the point of intersection of **PS** and **QR.** Show that K divides **PS** in the ratio 1:3. *(12 marks)*

15. (a) Find the equation of plane that contains the points A(1,2,3) and B(2,-1,2) and parallel to the line $\frac{x-2}{1} = \frac{2-y}{-2} = \frac{z-3}{3}$. *(05 marks)*

(b) A right circular cone has its vertex at the point (2,1,3) and the centre of its plane face at the point (1,-1,2). The generator of the cone (slanting side) has equation $\boldsymbol{r} = (2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k}) + t(\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k})$. Find the radius of the cone. *(07 marks)*

16. (a) Show that the shortest distance from the plane with equation $ax + by + cz = d$ to the point $A(p, q, r)$ is given by $\frac{|ap+bq+cr-d|}{\sqrt{a^2+b^2+c^2}}$.

(05 marks)

(b) Obtain the distance from the origin to the plane $2x - 6y + 3z = 14$.

(03 marks)

(c) Calculate the distance between the planes $2x - 6y + 3z = 14$ and $4x - 12y + 6z + 21 = 0$. *(04 marks)*

GOOD LUCK

MATHEMATICS HOLIDAY PACKAGE

INSTRUCTIONS

- Answer ***ALL*** questions in Section A and B.
- Any additional question(s) answered will not be marked.
- All necessary calculations must be done in the answer booklet provided. Therefore no papers should be given for rough work.
- Graph paper is provided.
- Silent non programmable scientific calculators and Mathematical tables with list of formulae may be used.

SECTION A

1. Simplify: $\left(\frac{16}{81}\right)^{\frac{-1}{4}}$ (04mks)

2. Find the equation of the line passing through the point $P(5, 9)$ and parallel to the line joining the point $Q(15, -2)$ to point $R(-3,4)$. (04mks)

3. Musa bought a car at a discount of 5%. The market price of the car was 24,000,000/=. How much did he buy the car. (04mks)

4. Given that $R(2, 3)$and $S(5, 8)$ are two points in a plane, determine the;
   (a) vector $\boldsymbol{RS}$
   (b) magnitude of $\boldsymbol{RS}$ (04mks)

5. Solve the equation: $\log_{10}(7y + 2) - \log_{10}(y - 1) = 0$ (04mks)

6. In a class of 30 students, 15 liked Mathematics, 18 liked English and 4 liked neither Mathematics nor English. Find the number of students who like both Mathematics and English. (04mks)

7. The function $f(x) = ax^2 + 4x$. If $f(-1) = 3$. Find the value of a. (04mks)

8. The capacity of a cylindrical tin is 2 litres. Its radius is $8cm$, find its height. (04mks)

9. Express $2.6363\ldots\ldots\ldots\ldots\ldots$as a fraction in its simplest form. (04mks)

10. The scale on a map is $1:2000$. A building is represented on a map by an area of $3cm$. Find the actual areas in cm^2 occupied by building. (04mks)

SECTION B

11. If $h(x) = bx + 3$ and $h(4) = 23$
   (a) find the value of;
      (i) b
      (ii) $h(0)$
      (iii) $h(-5)$ (07mks)

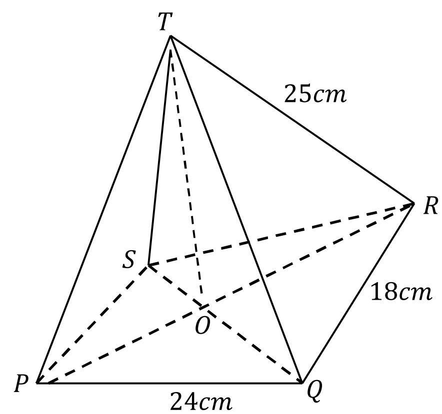
   (b) determine
      (i) $h^{-1}(x)$
      (ii) $h^{-1}(13)$ (05mks)

12. A quantity x is partly constant and partly varies as the square of y. When $y = 2, x = 40$, when $y = 3,\ P = 65$.
   (a) form an equation connecting x and y. (08mks)
   (b) determine y when x is 100. (04mks)

13. In a class of 40 students, 18 play Hokey (H), 15 play Tennis (T) and 22 play Football (F). 7 play Hockey and Tennis, 9 play Tennis and Football, 8 play Hockey and Football. 4 play all the three games.
   (a) Represent the given information on a venn diagram (06mks)
   (b) Find the number of students who do not play any of the three games. (02mks)
   (c) Find the probability that a student picked at random plays only:
      (i) one game
      (ii) two games (04mks)

14. A cyclist sets off from town A at $4:00\ am$ at a speed of $20km/h$r to go to town B $100km$ away. A motorist sets off from town A at $7:30\ am$ at a speed of $100km/hr$ to go town B. Find the:
   (a) distance from A when the motorist over takes the cyclist. (06mks)
   (b) the time when the motorist over takes the cyclist. (03mks)
   (c) time the cyclist reached B. (03mks)

15. In the figure below; $PQRS$ is a right pyramid with a rectangular base $PQ = 24cm$, $QR = 18cm$. The slanting edges are $25cm$ each.



Calculate the:

(a) Height of the pyramid. (06mks)

(b) Angle between slanting face QRT and the base. (03mks)

(c) The volume of the pyramid. (03mks)

16. Given that the point $A(-8, 6)$ and vector $\boldsymbol{AB} = \begin{pmatrix} 12 \\ 4 \end{pmatrix}$, M is the midpoint of AB.

(a) Find the:

(i) column vector $\boldsymbol{AM}$

(ii) coordinates of M

(iii) magnitude of $\boldsymbol{OM}$ (08mks)

(b) Draw the vector $\boldsymbol{AB}$ on a graph paper from your graph, state the coordinates of B. (04mks)

MY EFFORT MY FUTURE

PROBABILITY REVISION QUESTIONS

1. A discrete random variable Y has the probability distribution function given by;

y	5	8	9	11	12
$P(Y=y)$	a	0.1	a	0.4	0.1

Find the; (i) value of a (ii) E(5Y-7) (iii) Var(3Y)

2. A discrete random variable X has a pdf, f(x) given by; $f(x) = \begin{cases} k(x+1), & x = 1,2,3,4 \\ kx, & x = 5,6,7 \\ 0, & elsewhere \end{cases}$

   Find the;
   a) value of a constant, k
   b) $P(2 \leq x \leq 7)$
   c) mean and mode of X
   d) standard deviation of X
   e) semi inter-quartile range

3. Awilo played 15 chess games. The probability that she wins a game is 0.6.
   (i) Find the probability that she won exactly 6 games
   (ii) Calculate the most likely number of games she won.

4. A random variable X has the following probability distribution

   P(X = -2) = P(X = -1) = 2P(X= 0), P(X = 1) = 0.2, 2P(X = 2) = P(X =3)= 0.3

   Find the;

   a. **P(X = 2)** and **P(X = 0)**
   b. **P(x ≤2/x ≥0)**
   c. standard deviation of X
   d. upper quartile

5. A student answers ***3*** questions**.** The chance of getting each question correct is $\boldsymbol{\frac{2}{3}}$**.** Find the**:**

   (i) probability distribution for the number of correct answers
   (ii) probability of obtaining at most ***2*** correct answers**.**

6. A based coin is such that a head is three times as likely to occur as a tail, if its tossed five times. Find the probability that
   (i) at least two heads occur
   (ii) exactly three tails occur.

7. A and B are two independent events with A thrice as likely to occur as B. Find
   (i) *P(A∪B)* *(ii)* $P((A/\bar{B})$

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8. Two events A and B are independent such that **P(A U B) = 5 P(B) = 4P(A).**
   Determine P(A) and P(A n B)

9. Events A and B are such that $P(A) = \frac{4}{7}$, $P(A \cap \bar{B}) = \frac{1}{3}$ and $P(A/B) = \frac{5}{14}$ Find
   (i) P(A∩B) (ii) *P(B)*

10. *A* and *B* are two independent events with A twice as likely to occur as *B*. If $P(A) = ½$, find
    (i) $P(A \text{ or } B \text{ but not both})$ (ii) $P\left(A/_{B'}\right)$

11. In a certain inter university tournament; **35%** watched football but not cricket, **10%** watched cricket but not football and **40%** did not watch either game. Find the probability that they watched football, given that they did not watch cricket.

12. The probability that it will be sunny tomorrow is 0.25. The probability that Mrs Black will go shopping tomorrow is 0.6. The probability that both these events occur is 0.15. Find the probability that;

    (i) Mrs Black goes shopping given that it is not sunny.
    (ii) it is sunny given that Mrs Black goes shopping.

13. A certain mechanism consists of the three components and will operate properly only if all three components are functioning. Assuming that the three components function independently of one another and have probabilities 0.02, 0.05 and 0.10 respectively of developing a fault.
    (a) Find the probability that the mechanism will not operate properly
    (b) Given that the mechanism is not operating, find the probability that it is because exactly one of the components have developed a fault.

14. *A* and *B* are mutually exclusive events such that $P(A) = 0.5$ and $P(B) = 0.4$.
    Find
    (i) $P(A' \cap B')$ (ii) $P(A' \cup B)$

15. Mark, Mary and Mable applied for a job in a company. Their respective probabilities of getting a job were 0.8, 0.7 and 0.6. find the probability that
    (a) none of them got a job
    (b) at least one of them got a job.

16. (a) Given that A and B are two events such that $P(A')=0.3$, $P(B) = 0.1$ and $P(A/B') = 0.2$.
Find (i) $P(A \cup B)$ (ii) $P((A/B')$

(b) A box contains 1 red, 3 green and 1 blue ball. Box B contains 2 red, 1 green and 2 blue balls. A balanced die is thrown and if the throw is a six, box A is chosen, otherwise box B is chosen. A ball is drawn at random from the chosen box. Given that a green ball is drawn, what is the probability that it came from box A?

17. Three machines A, B and C produce solar bulbs in the ratio 30%, 60% and 10%. Of those produced by machine A, 25% are coloured, that of B is 30% and that of C is 70%. Find the probability that a bulb selected at random is;
 (i) coloured
 (ii) produced by C given that is not coloured.

18. A random variable t takes values in the interval $0 < t < 3$ and has probability density function f(t) given by

$$f(t) = \begin{cases} at; \ 0 \leq t \leq 1 \\ \frac{a}{2}(3-t); \ 1 \leq t \leq 3 \\ 0 \ ; \ elsewhere \end{cases}$$

Where a is a constant, find the
 (i) value of a
 (ii) expected value and variance of T
 (iii) determine the distribution function of T, hence find $P(|T-1| < \frac{1}{2})$

19. The probability density function of a random variable X is given by;

$$f(x) = \begin{cases} \frac{4}{5}x & ; 0 < x < 1 \\ \frac{2}{5}(3-x) & ; 1 < x < 2 \\ 0 & ; \text{otherwise} \end{cases}$$

 (a) Sketch the function f(x) and show that the area = 1
 (b) Find the mean of x
 (c) Determine the cumulative distribution function $F(x)$

20. A probability density function is given as;

$$f(x)= \begin{cases} k\,x\,(4-x^2); & 0\leq x \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

Find (i) the value of k
 (ii) mode
 (iii) mean
 (iv) standard deviation

END

ST JOSEPHS SEMINARY

P425/2

S6 APPLIED MATHEMATICS   REVISION QUESTIONS ON

NUMERIAL METHODS   2022

1(a)Use the trapezium rule with six ordinates to estimate $\int_0^2 xsinx^2dx$ correct to four decimal places.

(b)Calculate the exact value of $\int_0^2 xsinx^2dx$ correct to four decimal places.

(c)Calculate the percentage error made in your calculation in (a) and state how the error can be reduced.

2 The numbers x and y are approximated by X and Y with errors Δx and Δy respectively

(a) Show that the maximum relative error in $\frac{y^2}{x}$ is given by; $\left|\frac{\Delta x}{x}\right|+2\left|\frac{\Delta y}{y}\right|$

(b) If x=6.75 and y=4.285 are each rounded off to the given number of decimal places ,calculate the;

(i) Percentage error in $\frac{y^2}{x}$

(ii) Limits within which $\frac{y^2}{x+y}$

3(a) Find the exact value of $\int_1^{1.8} tan\frac{1}{2}x dx$. Correct to three decimal places.

(b)use the trapezium rile with seven ordinates to evaluate; $\int_1^{1.8} tan\frac{1}{2}xdx$.Correct to three decimal places.

(c) Calculate the percentage error made in your answer in (b) above. Suggest how the error may be reduced.

4(a) show by drawing graphs of the y=1-e^x and y=2^x, that the equation 2^x=1-e^x has a root between -1.5 and 0.Hence find the root to 2 decimal places.

(c) Using NEWTON RAPHSON METHOD, show that the seventh root of a number K is $x_{n+1}=\frac{6x_n^7 + K}{7x_n^6}$, Hence, if K=66 and the initial approximation of the root is 1.9,find the root correct to three significant figures.

5(a) The numbers x and y are approximated with errors Δx and Δy respectively. Deduce that the maximum error made in estimating x^2y is given by 2$|xy\Delta x|$+$|x^2y|$. Hence find the limits within which the exact value of 2.4^2x 0.18 lies.

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(b) The relative error in measuring the volume of a cylinder is 0.125 and that in measuring the height is 0.05. Calculate the percentage error made in measuring the radius.

6(a) Show that Newtons Raphson iterative formula for solving the equation $2x^3$+5x-8=0 is

x_{n+1} $=\frac{4x_n^3+8}{6x_n^2+5}$.

(b) Taking the first approximation to the largest positive root as 1.4, draw a flow chart diagram which reads and prints the number of iterations and the root with an error of less than 0.001. Carry out a dry run for the flow chart.

7(a) Show that the iterative formular based on Newton Raphson Method for solving the root of the equation e^{2x}+ 4x =5 is given by;

x_{n+1}=$\frac{e^{2x_n}(2x_n-1)+5}{2e^{2x_n}+4}$,n=0,1,2,3……………………

(b)

(i) Construct a flow chart that:

-reads the initial approximation x_0,

-computes, using the iterative formula in (a) and prints the root of the equation e^{2x}+4x=5, and the number of iterations when the error is less than 1.0X10^{-4}

(ii) perform a dry run for the flow chart when x_0=0.5.

8(a) The table below shows the values of x and their corresponding natural logarithm.

X	5.0	5.2	5.4	5.7	6.0
Inx	1.609	1.647	1.686	1.740	1.792

Use linear interpolation or extrapolation to find

(i) In (5.56)
(ii) $e^{1.575}$

(b) A car consumed fuel amounting to shs 14,800,shs 15,600 and shs 17,200 in covering distances of 10km,20km, 30km and, 40km respectively. Estimate the;

(i) cost of fuel consumed for a distance of 45km,

(ii) distance travelled if fuel of shs 16,000 is used.

9(a) Using trapezium rule with five strips evaluate $\int_3^4 \frac{1}{\sqrt{(x-1)^2-3}} dx$, correct to three decimal places.

(b) Find the exact value of $\int_3^4 \frac{1}{\sqrt{(x-1)^2-3}} dx$

© Find the percentage error in the approximation in (a) above and suggest how the error can be reduced.

10(a) Given that y=xsinx and x=2, find the absolute error in y giving your answer correct to three significant figures.

(b)The numbers x=1.5, y=-2.85 and z=10.345 were all rounded off to the given number of decimal places. Find the range within which the exact value of $\frac{1}{x}$ - $\frac{1}{y}$ + $\frac{y}{xz}$ lies.

11(a) Two positive decimal numbers X and Y were approximated with errors E_1 and E_2 respectively .Show that the maximum positive relative error in the approximation of the product X^3Y^2 is 3$\left|\frac{E_1}{X}\right|$+2$\left|\frac{E_2}{Y}\right|$.

(b) Given that X=5.64 and Y=10.0, rounded off to the given number of decimal places .Find the;

(i)maximum possible errors in X and Y.

(ii) percentage error made in the approximation of X^3Y^2.

12 (a) Use trapezium rule with five strips to estimate $\int_0^4 3^{2x}\, dx$ correct to 2 decimal places.

(b) Find the exact value of $\int_0^4 3^{2x} dx$ correct to 2 decimal places.

(c) Calculate the relative error made in (a) above.

13(a) Find the maximum possible error made in the expression 6.23 - 3.1 - $\frac{2.5\times4.1}{5}$ correct to three decimal significant figures.

14(a) The numbers x=3.7 and y=70 are each rounded off with percentage errors of 0.2 and 0.5 respectively. While z is calculated with relative error of 0.04.Find the interval within which the exact value of $\frac{x}{y-z}$ lies; correct your answer to 4 significant figures.

(b) The height and radius of a cylindrical water tank are given as H=3.5$\pm$0.2 and R=1.4$\pm$0.1 respectively.Determine in m^3, the least and greatest amount of water the tank can contain. Hence, calculate the maximum possible error in your calculation.

Examination questions:

1. The r th term of an arithmetic series is $(2r - 5)$.

(a) Write down the first three terms of this series and state the value of the common difference.

(b) Show that $\sum_{r=1}^{n}(2r - 5) = n(n - 4)$.

2. (a) The terms of an arithmetic sequence are given by $u_n = (n+k)^2$, $n \geq 1$, where k is a positive constant. Given that $u_2 = 2u_1$, find the value of k and show that $u_1 = 11 + 6\sqrt{2}$.

(b) Prove that;

$\log a + \log ax + \log ax^2 + \cdots \ldots \ldots to\ n\ terms = n \log a + \frac{1}{2}n(n-1)\log x$.

3. The 5th and 12th terms of an AP are 2 and 23 respectively. Find:

(a) The 9th term.

(b) The sum of the first 20 terms.

4. There are 20 terms of an AP. The sum of the first 10 terms is 55 and the sum of the last 10 terms is 355. Find the first term and the common difference.

5. If the 9th term of an AP is 3 times the 3rd term and the sum of the first 10 terms is 110, find the first term and the common difference.

6. In an AP the sum of the first three terms is 12 and their product is 28. Find the possible values of the first term and the common difference.

7. The numbers $x+3$, $5x+3$ and $11x+3$ $(x \neq 0)$ are three consecutive terms of a GP. Find the value of x and the common ratio.

8. (a) The sum of the first 8 terms of an arithmetic progression is 24 and the sum of the first 18 terms is 90. Calculate the value of the seventh term.

(b) A geometric progression with a positive common ratio is such that the sum of the first 2 terms is $17\frac{1}{2}$ and the third term is $4\frac{2}{3}$. Calculate the value of the common ratio.

9. (a) An arithmetic progression contains 20 terms. Given that the 8th term is 25 and the sum of the last 8 terms is 404, calculate the sum of the first 10 terms.

(c) The first term of a geometric progression exceeds the second term by 2, and the sum of the second and the third term is $\frac{4}{3}$.

Calculate the possible values of the first term and the common ratio of the progression. Given further that all the terms of the progression are positive, calculate the sum to infinity.

10. (a) The first term of an arithmetic progression is 4. The sum of the first 10 terms is 100 and the sum of the whole series is 136.

Calculate (i) the common difference, (ii) the number of terms, (iii) the last term.

(b) Find the sum of the integers between 50 and 150 which are divisible by 8.

11. The fourth term of an arithmetic series is $3k$, where k is a constant , and the sum of the first six terms of the series is $7k + 9$.

(a) Show that the first term of the series is $9 - 8k$.

(b) Find an expression for the common difference of the series in terms of k.

Given that the seventh term of the series is 12, calculate:

(c) The value of k.

(d) The sum of the first 20 terms of the series.

12. (a) Evaluate:

$$\sum_{r=1}^{75}(1-2r^2).$$

(b) Show that $\sum_{r=1}^{n}\frac{r+3}{2}=kn(n+7)$ where k is a rational constant to be found.

13. The first two terms of an arithmetic series are $(x-2)$ and (x^2+4) respectively, where x is a positive constant.

(a) Given also that the third term of the series is 20, find the value of x.

(b) Given that the sum of n terms of the series is 277. Find the n

14. A geometric series has first term a and common ratio r. The second term of the series is 4 and the sum to infinity of the series is 25.

(a) Show that $25r^2-25r+4=0$ and find the two possible values of r and a.

(b) Show that the sum S_n of the first n terms of the series is given by $S_n=25(1-r^n)$.

(c) Given that r takes the larger of the two possible values, find the smallest value of n for which S_n exceeds 24.

15. The first three terms of a geometric series are $(p-2)$, $(p+6)$ and p^2 respectively.

(a) Show that p must be a solution of the equation $p^3-3p^2-12p-36=0$.

(b) Verify that $p=6$ is a solution of equation and show that there are no other real solutions.

(c) Using $p=6$, find the common ratio of the series, and the sum of the first six terms of the series.

16. The second and fifth terms of a geometric series are -48 and 6 respectively.

(a) Find the first term and the common ratio of the series.

(b) Find the sum to infinity of the series.

(c) Show that the difference between the sum of the first n terms of the series and its sum to infinity is given by 2^{6-n}.

17. The second and third terms of a geometric series are $\log_3 4$ and $\log_9 256$ respectively.

(a) Show that the common ratio of the series is 2.

(b) Show that the first term of the series is $\log_3 2$.

(c) Find, to 1 decimal place, the sum of the first five terms of the series.

18. The sum of the first three terms of a geometric series is 210. The sum to infinity of the series is 480.

(a) Find the two possible values of the common ratio and first term.

(b) Given that r is positive and that the sum of the first n terms of the series is greater than 300. Calculate the smallest possible value of n.

19. An arithmetic series has first term a and common difference d.

(a) Prove that the sum of the first n terms of the series is $\frac{1}{2}n[2a + (n - 1)d]$.

Sean repays a loan over a period of n months. His monthly repayments form an arithmetic sequence. He repays \$149 in the first month, \$147 in the second month, \$145 in the third month, and so on.

(b) Find the amount Sean repays in the 21^{st} month.

Over the n months, he repays a total of \$5,000.

(c) Form an equation in n, and show that your equation may be written as $n^2 - 150n + 5000 = 0$. Find n.

(d) State, with a reason, which of the solutions to the equation in part (c) is not a sensible solution to the repayment problem.

20. On Alice's 11th birthday she started to receive an annual allowance. The first annual allowance was \$500 and on each following birthday the allowance was increased by \$200.

(a) Show that, immediately after her 12^{th} birthday, the total of the allowance that Alice had received was \$1200.

(b) Find the total of the allowances that Alice had received up to an including her 18^{th} birthday.

(c) When the total of the allowances that Alice received reached \$32,000 the allowance stopped. Find how old Alice was when she received the last allowance.

21. An athlete prepares for a race by completing a practice run on each of the 11 consecutive days. On each day after the first day he runs further than he ran on the previous day. The lengths of his 11 practice runs form an arithmetic sequence with first term a km and common difference d km.

He runs 9 km on the 11^{th} day, and he runs a total of 77 km over the 11 day period. Find the value of a and the value of d.

22. (a) Prove that the sum of the first n terms of an arithmetic series with first term a and common difference d is given by $\frac{1}{2}n[2a + (n - 1)d]$.

A novelist begins writing a new book. She plans to write 16 pages during the first week, 18 during the second and so on, with the number of pages increasing by 2 each week. Find, according to her plan:

(a) How many pages she will write in the fifth week.

(b) The total number of pages she will write in the first five weeks.

(c) Using algebra, find how long it will take her to write the book if it has 250 pages.

23. As part of a new training program, Habib decides to do sit-ups everyday. He plans to do 20 per day in the first week, 22 per day in the second week, 24 per day in the third week and so on, increasing the daily number of sit-ups by two at the start of each week.

(a) Find the number of sit-ups that Habib will do in the fifth week.

(b) Show that he will do a total of 1512 sit-ups during the first eight weeks.

(c) In the nth week of training, the number of sit-ups that Habib does is greater than 300 for the first time. Find the value of n.

24. (a) A geometric series has first term a and common ratio r. Prove that the sum of the first n terms of the series is

$$\frac{a(1-r^n)}{1-r}.$$

Mr. King will be paid a salary of \$35 000 in the year 2005. Mr. King's contract promises a 4% increase in salary every year, the first increase being in 2006, so that his annual salaries form a geometric sequence.

(b) Find to the nearest \$100, Mr. King's salary in the year 2008.

(c) Mr. King will receive a salary each year from 2005 until he retires at the end of 2024. Find to the nearest \$1000, the total amount of salary he will receive in the period from 2005 until he retires at the end of 2024.

Examination Questions

1. (*a*) $(-3) + (-1) + 1;\ d = 2$

   (*b*) $\sum_{r=1}^{n}(2r - 5) = n(n - 4)$
2. (*a*) $k = \sqrt{2}$
3. (*a*) $U_9 = 14$ (*b*) $S_{20} = 370$
4. $(a = -8, d = 3)$
5. $(a = 2, d = 2\ )$
6. $(d = -3, a = 7)$ *or* $(d = 3, a = 1)$
7. $\left(x = \frac{3}{7}, r = \frac{3}{2}\right)$
8. (*a*) $U_7 = 4$ (*b*) $r = \frac{2}{3}$
9. (*a*) $S_{10} = 175$ (*b*) $\left(r = \frac{1}{3}, a = 3\right)$ *or* $\left(r = -2, a = \frac{2}{3}\right)$; $S_\infty = \frac{9}{2}$
10. (*a*) (i) $d = \frac{4}{3}$ (ii) $n = 12$ (iii) $\frac{56}{3}$

    (*b*) 1200
11. (*a*) $a = 9 - 8k$ (*b*) $d = \frac{1}{3}(11k - 9)$ (*c*) $k = \frac{3}{2}$ (*d*) $S_{20} = 415$
12. (*a*) -16725

    (*b*) $\frac{1}{4}n(n + 7);\ k = \left(\frac{1}{4}\right)$
13. (*a*) $x = \frac{5}{2}$ (*b*) $n = 8$
14. (*a*) $\left(r = \frac{1}{5}, a = 20\right)$ *or* $\left(r = \frac{4}{5}, a =\right.$ *5 b n=15*
15. (*c*) $r = 3;\ S_6 = 1456$
16. (*a*) $a = 96;\ r = -\frac{1}{2}$ (*b*) $S_\infty = 64$
17. (*c*) 20.2
18. (*a*) $\left(r = -\frac{3}{4}, a = 840\right)$ *or* $\left(r = \frac{3}{4}, a =\right.$ *120 b n=4*
19. (*b*) \$109 (*c*) $n = 100$ *or* $n = 50$

    (*d*) $U_{100} < 0;\ \therefore n = 100$ *is not sensible.*
20. (*b*) \$9600 (*c*) 26 *yrs*
21. $a = 5;\ d = \frac{2}{5}$
22. (*a*) 24 *pages* (*b*) 100 *pages* (*c*) 10 *weeks*
23. (*a*) 196 *sit ups* (*c*) $n = 13$
24. (*b*) 39400 (*c*) 1,042,000

Examination Questions

1. (a) Show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

   Hence solve the equation $4x^3 - 3x - \frac{\sqrt{3}}{2} = 0$.

   (b) Find all the solutions of the equation $4\cos x - 5\sin x = 6$ in the range $0° \leq x \leq 360°$.

2. Given that $\sin x + \sin y = \alpha_1$ and $\cos x + \cos y = \alpha_2$, show that:

   (a) $\tan\left(\frac{x+y}{2}\right) = \frac{\alpha_1}{\alpha_2}$

   (b) $\cos(x+y) = \dfrac{\alpha_2^2 - \alpha_1^2}{\alpha_2^2 + \alpha_1^2}$.

   (c) Solve the simultaneous equations for values of x and y between $0°$ and $360°$:

$$\cos x + 4\sin y = 1$$

$$4\sec x - 3\csc y = 5$$

3. (a) Given that $7\tan\theta + \cot\theta = 5\sec\theta$, derive a quadratic equation for $\sin\theta$. Hence or otherwise, find all values of θ in the interval $0° \leq \theta \leq 180°$ which satisfy the given equation, giving your answers to the nearest $0.1°$ where necessary.

(b) The acute angle A and B are such that $\cos A = \frac{1}{2}, \sin B = \frac{1}{3}$. Show without using tables or calculator that;

$$\tan(A + B) = \frac{9\sqrt{3} + 8\sqrt{2}}{5}.$$

4. (a) Prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

(b) Find all the solutions to $2\sin 3\theta = 1$ for θ between $0°$ and $360°$. Hence find the solutions of $8x^3 - 6x + 1 = 0$.

5. (a) Show that $x = 1$ is a solution of the equation $x^3 - x^2 - 3x + 3 = 0$, and find the other two values of x which satisfy the equation.

(b) Use part (a) to show that $\tan\theta = 1$ is a solution of the equation $\tan^3\theta - 3\tan\theta + 4 = \sec^2\theta$. And hence find all the values of θ satisfying the equation $(0° \leq \theta \leq 360°)$.

6. (a) Using the formulae for $\sin(A \pm B)$ and $\cos(A \pm B)$, show that;

$$\frac{\cos(A - B) - \cos(A + B)}{\sin(A + B) - \sin(A - B)} = \tan A .$$

(b) Using the result of (a) and the exact values of $\sin 60°$ and $\cos 60°$, find an exact value of $\tan 75°$ in its simplest form.

7. (a) Prove that $\sin x + \cot x \cos x = \csc x$.

(b) Hence or otherwise, find the values of x, $0° < x < 180°$, which satisfy the equation $\cot x \cos x = 3$, giving your answer to 1 decimal place.

8. (a) Express $f(x) = \sqrt{3}\sin x + \cos x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Hence solve the equation $\sqrt{3}\sin x + \cos x = \sqrt{2}$ where $0° < x < 180°$.

(b) Sketch the graph of $y = f(x)$ for $0° \leq x \leq 360°$.

(c) You are given that $y = 2f(x) + 1$. State the maximum and minimum values of y and the values of x when they occur.

9. (a) Prove that;

$$\cot 2\theta = \frac{\cot^2\theta - 1}{2\cot\theta}.$$

(b) Use the identity to find the values of θ, for $0° < \theta < 360°$, which satisfy the equation $\cot^2\theta - 2\cot\theta - 1 = 0$.

10. (a) Express $7\sin x + 24\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Hence solve the equation $7\sin x + 24\cos x = 15$, where $0° < x < 360°$.

(b) Prove that these values satisfy the equation $15\sec x - 7\tan x = 24$.

(c) Find the maximum value of the function $7\sin x + 24\cos x$ and give the smallest positive value of x for which this maximum value occurs.

11. (a) Express $2.5\sin 2x + 6\cos 2x$ in the form $R\sin(2x+\alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving your values of R and α to 2 decimal places.

(b) Express $5\sin x\cos x - 12\sin^2 x$ in the form $a\cos 2x + b\sin 2x + c$, where a, b and c are constants to be found.

(c) Hence using your answer to part (a), deduce the maximum value of

$5\sin x\cos x - 12\sin^2 x$ and the value of x when it occurs.

12. (a) Given that $\cos(2x-60) = 2\sin(2x+30)$, prove that $\tan 2x = -\frac{1}{\sqrt{3}}$.

(b) Using the result from part (a), find the two values of x, $0° < x < 180°$, which satisfy the equation $2\sin(2x+30) - \cos(2x-60) = 0$.

13. (a) Express $9\cos\theta - 40\sin\theta$ in the form $R\cos(\theta+\alpha)$ where $R > 0$ and $0° < \alpha < 90°$.

Hence solve the equation $9\cos\theta - 40\sin\theta = 6$, for $0° < \theta < 90°$, giving your answer to 1 decimal place.

(b) Solve the equation $13 + 10\cot\theta = 3\tan\theta$, for $0° < \theta < 180°$, giving your answer to 1 decimal place.

14. (a) Letting $A+B=P$, and $A-B=Q$ and using the expansion for $\sin(A \pm B)$, prove that

$$\sin P - \sin Q = 2\cos\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$$

(b) Hence or otherwise solve the equation;

$\sin 4\theta - \sin 2\theta + \cos 3\theta = 0$ for $0° < \theta < 360°$.

15. (a) Prove that;

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}.$$

(b) Hence solve the equation $\tan\theta\,(4-\tan\theta) = 1,\ 0° < \theta < 360°$.

16. (a) Using the identity for $\cos(A+B)$, prove that $\cos\theta = 2\cos^2\left(\frac{1}{2}\theta\right) - 1$.

(b) Prove that $1 + \sin\theta + \cos\theta = 2\cos\left(\frac{1}{2}\theta\right)\left[\sin\left(\frac{1}{2}\theta\right) + \cos\left(\frac{1}{2}\theta\right)\right]$

(c) Hence, or otherwise, solve the equation $1 + \sin\theta + \cos\theta = 0,\ 0° \le \theta \le 360°$.

17. (a) Find the solution of the equation $\tan x + \sec x = 3\cos x$ for $0° \le x \le 360°$.

(b) Express $5\sin^2 x - 3\sin x\cos x + \cos^2 x$ in the form $a + b\cos(2x-\alpha)$ where a, b and α are independent of x. Hence or otherwise, find the maximum and minimum values of $5\sin^2 x - 3\sin x\cos x + \cos^2 x$ as x varies.

18. (a) Express $\frac{\sin 2\theta - \cos 2\theta - 1}{2 - 2\sin 2\theta}$ in terms of $\tan\theta$.

(b) Solve $\sin 3x + \frac{1}{2} = 2\cos^2 x$ for $0° \le x \le 360°$.

19. (a) Prove that:

$$(\sin 2\theta - \sin\theta)(1 + 2\cos\theta) = \sin 3\theta.$$

(b) A vertical pole BAO stands with its base O on a horizontal plane, where $BA = c$ and $AO = b$. A point P is situated on a horizontal plane x $units$ from O and the angle $APB = \theta$.

Prove that $\tan\theta = \frac{cx}{x^2+b^2+bc}$.

20. (a) Prove that $\sin 2x = \frac{2\tan x}{1+\tan^2 x}$.

(b) Solve for x in:

(i) $\tan x + 3\cot x = 4$

(ii) $4\cos x - 3\sin x = 2;\ \ 0° \le x \le 360°$.

21. (a) Given that X, Y, and Z are angles of a triangle XYZ. Prove that $\tan\left(\frac{X-Y}{2}\right) = \frac{x-y}{x+y}\cot\frac{Z}{2}$. Hence solve the triangle if $x = 9\ cm$, $y = 5.7\ cm$, and $Z = 57°$.

(b) Use the substitute $t = \tan\frac{\theta}{2}$ to solve the equation $3\cos\theta - 5\sin\theta = -1$ for $0° \le \theta \le 360°$.

22. (a) Show that $\cos 4\theta = \frac{\tan^4\theta - 6\tan^2\theta + 1}{\tan^4\theta + 2\tan^2\theta + 1}$.

(b) Given that in any triangle ABC

$$\tan\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{b+c}\right)\cot\left(\frac{A}{2}\right);$$

solve the triangle with two sides 5 cm and 7 cm and the included angle 45°.

23. (a) Solve the equation $3\cos x + 4\sin x = 2$ for $0° \le x \le 36°$.

(b) If A, B, C are angles of the triangle, show that $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A\cos B\cos C$.

24. (a) Show that $\frac{\sin\theta - 2\sin 2\theta + \sin 3\theta}{\sin\theta + 2\sin 2\theta + \sin 3\theta} = -\tan^2\frac{\theta}{2}$.

(b) Express $4\cos\theta - 5\sin\theta$ in the form $R\cos(\theta + \beta)$, where R is a constant and β an acute angle.

(c) Determine the maximum value of the expression and the value of θ for which it occurs.

(d) Solve the equation $4\cos\theta - 5\sin\theta = 2.2$, for $0° < \theta < 360°$.

25. (a) Find all the values of θ, $0° \le \theta \le 360°$ which satisfy the equation $\sin^2\theta - \sin 2\theta - 3\cos^2\theta = 0$.

(b) Show that $\frac{\cos A}{1+\sin A} = \cot\left(\frac{A}{2} + 45°\right)$. Hence or otherwise solve $\frac{\cos A}{1+\sin A} = \frac{1}{2}$; $0° \le A \le 360°$.

26. (a) Express $\cos\theta + 2\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(b) Find the maximum and minimum values of $\cos\theta + 2\sin\theta$ and the smallest possible value for θ for which the maximum occurs.

The depth d $metres$, of water in a lake is modeled using the equation where t $hours$ is the number of hours after 1200

$$d = 15 + \cos\left(\frac{\pi t}{12}\right) + 2\sin\left(\frac{\pi t}{12}\right),\ \ 0 \le t \le 24.$$

(c) Calculate the maximum depth of the water predicted by this model and the value of t when this maximum occurs.

(d) Calculate the depth of the water at 1200.

(e) Calculate to the nearest half hour the time in the evening when the depth of the water is 15 metres.

Examination Questions

1. (*a*) $x = -0.643, -0.342, 0.985$
   (*b*) $x = 288.2, 329.0$

2. (*c*) $x = 78.5, 281.5$; $y = 11.5, 168.5$

3. (*a*) $\theta = 19.5, 30, 150, 160.5$

4. (*b*) $\theta = 10, 50, 130, 170, 250, 290$; $x = 0.1736,\ 0.7660,\ -0.9397$

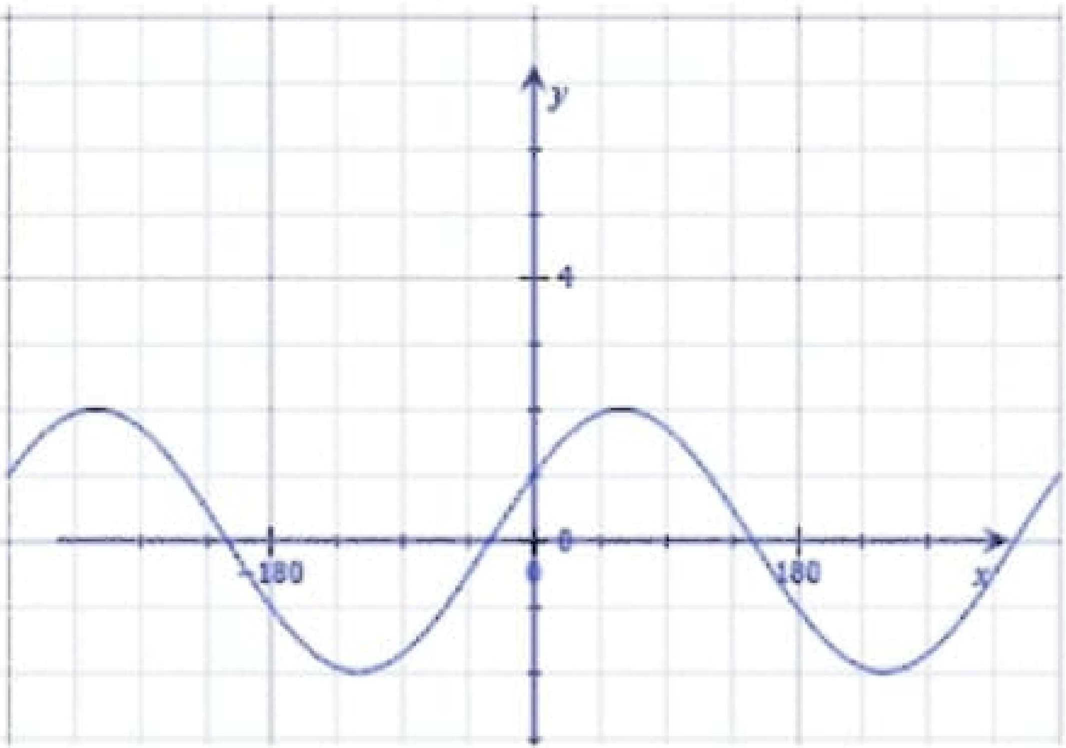
5. (*a*) $x = \sqrt{3}$ *or* $x = -\sqrt{3}$
   (*b*) $\theta = 45, 60, 120, 135, 240, 300$

6. (*b*) $2 + \sqrt{3}$

7. (*b*) $x = 17.6, 162.4$

8. (*a*) $2\cos(x - 60)$; $x = 105$

(*b*)



(c) $maxi = 5$ when $x = 60$; $mini = -3$ when $x = 240$

9. (b) $\theta = 22.5, 112.5, 202.5, 292.5$

10. (a) $25\sin(x + 73.7)$; $x = 69.4, 323.4$

    (c) $maxi = 25$ when $x = 16.3$

11. (a) $6.5\sin(2x + 67.38)$

    (b) $2.5\sin 2x + 6\cos 2x - 6$

14. (b) $\theta = 30, 90, 150, 210, 270, 330$

15. (b) $\theta = 15, 75, 195, 255$

16. (c) $\theta = 180, 270$

17. (a) $x = 41.8, 138.2, 270$

    (b) $mini = 0.5$; $max = 5.5$

18. (a) $\frac{1}{\tan\theta - 1}$

    (b) $x = 30, 60, 120, 150, 240, 300$

19. (b) $\tan\theta = \frac{cx}{x^2 + b^2 + bc}$

20. (b) $x = 45, 71.6, 225, 251.6$; $x = 29.6, 256.6$

21. (a) $X = 83.9°$, $Y = 39.1°$, $z = 7.6\,cm$

    (b) $\theta = 40.8, 201.0$

22. (b) $45.6°$, $89.4°$, $4.95cm$

23. (a) $x = 119.5, 299.5$

(c) $maxi = 0.5$ when $x = 11.13$

12. (b) $x = 75, 165$

13. (a) $41\cos(\theta + 77.3)$; $\theta = 4.3$

    (b) $\theta = 78.7, 146.3$

24. (b) $\sqrt{41}\cos(\theta + 51.3)$

    (c) $maxi = \sqrt{41}$ when $\theta = 308.7$

    (d) $\theta = 18.6, 238.8$

25. (a) $\theta = 71.6, 135, 251.6, 315$

    (b) $A = 36.8$

26. (a) $R = 2.24$, $\alpha = 1.11\,rad$

    (b) $\begin{pmatrix} maxi = \sqrt{5} \\ mini = -\sqrt{5} \end{pmatrix}$, $\theta = 1.11$

    (c) $maxi\ depth = 15 + \sqrt{5}$
    (17.24 m to 2dp); $t = 4.24$ (to 2dp)

    (d) $d = 16m$

    (e) $(t = 10.24)$,
    2230 to nearest half hr

P425/1

PURE MATHEMATICS

Paper 1

June/July. 2022

3 hours

TRIGONOMETRY TEST 2022

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

*Answer **all** the **eight** questions in section A and any **five** from section **B**.*

*Any additional question(s) answered will **not** be marked.*

***All** necessary working **must** be shown clearly.*

Begin each answer on a fresh page.

Silent non- programmable scientific calculators and mathematical tables with a list of formulae may be used.

TURN OVER

SECTION A: (40 MARKS)

*Attempt **all** questions in this section.*

1. Given that $cotA = \frac{4}{3}$ and $secB = \frac{17}{15}$ where A and B are both reflex angles. Find without using mathematical tables or calculator the value of $\tan(A - B)$. (05 marks)

2. Solve the equation: $8sin^2(\theta - 30^0) = 1 + cos2\theta$ for $0^0 \leq \theta \leq 360^0$.

   (05 marks)

3. Without using mathematical tables or calculator, prove that $cos165^0 + sin165^0 = cos135^0$. *(05 marks)*

4. Prove that $\frac{tanA+secA-1}{tanA-secA+1} = secA + tanA$. *(05 marks)*

5. Solve the equation: $3sinx + cos2x = 2$ for $-180^0 \leq x \leq 180^0$.

   (05 marks)

6. Solve the equation: $2cos^2\left(x - \frac{\pi}{2}\right) - 3cos\left(x - \frac{\pi}{2}\right) + 1 = 0$ for $0 \leq x \leq 2\pi$. *(05 marks)*

7. Solve: $sinx + sin2x + sin3x = 0$ for $0^0 \leq x \leq 180^0$. *(05 marks)*

8. Show that $\frac{sin3A}{1+2cos2A} = sinA$. Hence show that $sin15^0 = \frac{\sqrt{3}-1}{2\sqrt{2}}$.

   (05 marks)

SECTION B: (60 MARKS)

*Attempt only **five** questions in this section.*

9. (a) Show that if $tan\frac{\theta}{2} = t, sin\theta = \frac{2t}{1+t^2}$ and $cos\theta = \frac{1-t^2}{1+t^2}$. Hence solve the equation $3cosx - 5sinx = 2$ for $0^0 \leq x \leq 360^0$. *(07 marks)*

(b) Solve the equation: $5cos\theta sin2\theta + 4sin^2\theta = 4$ for $0^0 \leq \theta \leq 360^0$. *(05 marks)*

10.(a) Given $x = tan\theta - sin\theta, y = tan\theta + sin\theta$, show that $(x^2 - y^2)^2 - 16xy = 0$. *(05 marks)*

(b) Solve: $4sinxcos2xsin3x = 1$ for $0^0 \leq x \leq 180^0$. *(07 marks)*

11.(a) Show that $\frac{sin8\theta cos\theta - sin6\theta cos3\theta}{cos2\theta cos\theta - sin3\theta sin4\theta} = tan2\theta$. *(05 marks)*

(b)Given $ksinx = sin(x - \alpha)$, find $tanx$ in terms of k and α. Hence solve the equation $2sinx = sin(x - 60^0)$ for $0^0 \leq x \leq 360^0$.

(07 marks)

12.Express $12cos^2x - 16sinxcosx - 7$ in the form $a + bcos(2x + \alpha)$ where a and b are constants and α is a positive acute angle. Hence;

(a) Solve the equation: $12cos^2x - 16sinxcosx - 5 = 0$ for $0^0 \leq x \leq 360^0$. *(08 marks)*

(b) Find the maximum value of $\frac{1}{12cos^2x - 16sinxcosx - 5}$ and state the smallest positive value of x when it occurs. *(04 marks)*

13.(a) Show that if P,Q and R are angles of a triangle then $1 + cos2R - cos2P - cos2Q = 4sinPsinQcosR$. *(05 marks)* (b)Solve for x: $sin(x+30^0)+sin(x+60^0)=cos(x+45^0)+cos(x+75^0)$ for $0^0 \leq x \leq 360^0$. *(07 marks)*

14.(a) Prove that $tan(A-B)=\frac{tanA-tanB}{1+tanAtanB}$. Hence solve the equation $tan(\theta-45^0)=6tan\theta$ for $-180^0 \leq \theta \leq 180^0$. *(07 marks)*

(b)The acute angle α is such that $tan\left(\alpha+\frac{\pi}{4}\right)=41$, show that $cos\alpha=\frac{21}{29}$. Hence find $sin\alpha$. *(05 marks)*

15.(a) Prove that $sin4\theta=\frac{4tan\theta-4tan^3\theta}{(1+tan^2\theta)^2}$. Hence solve for t if $t=tan\theta$ given $t^4+8t^3+2t^2-8t+1=0$ correct to 3 significant figures.

(08 marks)

(b) Show that $\sqrt{\frac{1-sin\theta}{1+sin\theta}}=sec\theta-tan\theta$. *(04 marks).*

16.Prove that

(a) $sin^4\theta+cos^4\theta=\frac{1}{4}(3+cos4\theta)$. *(03 marks)*

(b) $cos^6x+sin^6x=1-\frac{3}{4}sin^2 2x$. *(04 marks)*

(c) $sin(A-B)+cos(A-B)tanC=sin2BsecC$ if A, B and C are angles of a triangle. *(05 marks)*

GOOD LUCK

TOPIC 4: FURTHER DIFFERENTIATION

The Chain rule

The expanding of some functions would be so complex for example $y=(2x+3)^{20}$. There is method which is used to differentiate such complex functions is known as chain rule.

Example

Differentiate $y = (3x+2)^{20}$

Let u = 3x +2

$$\frac{du}{dx} = 3$$

But $y = u^{20}$

$$\frac{dy}{du} = 20u^{19} = 20(3x+2)^{19}$$

$$\frac{dy}{du} = \frac{dy}{du}.\frac{du}{dx} = 20(3x+2)^{19}.3$$

$$=60(3x+2)^{19}$$

Example

Differentiate

$$y = \sqrt{x^2 - \frac{1}{x^2}}$$

Let $x^2 - \frac{1}{x^2} = u$

$$\frac{du}{dx}=(2x+\frac{2}{x^3})=$$

$$y=u^{\frac{1}{2}}$$

$$\frac{dy}{du}=\frac{1}{2}u^{-\frac{1}{2}}=y=\frac{1}{2\sqrt{u}}=\frac{1}{2\sqrt{x^2-\frac{1}{x^2}}}$$

$$=\frac{1}{2\sqrt{x^2-\frac{1}{x^2}}}.\left(2x+\frac{2}{x^2}\right)$$

$$=\frac{(2x^4+2)}{x^3}.\frac{1}{2\sqrt{\frac{x^4-1}{x}}}$$

$$=\frac{2x^4+2}{x^3}.\frac{x}{2\sqrt{x^4-1}}$$

$$=\frac{2(x^4+1)}{2x^2\sqrt{x^4-1}}$$

$$=\frac{(x^4+1)}{x^2\sqrt{x^4-1}}$$

Rates Of Change

The chain rule can also be used to investigate related rates of changes .

A learner is expected to identify this use the units given

Cubic units per time taken its $\frac{dv}{dt}$

Its square units per time taken its $\frac{dA}{dt}$ and

If it is units per time taken $\frac{dl}{dt}$

Example

A container in a shape of a right circular cone of height 10cmm and base radius 1cm is catching the drips from a tap leaking at a rate of $0.1cm^3s^{-1}$. Find the rate at which the surface area of water is increasing when the water is half way up. The cone .

Note that because of the units used

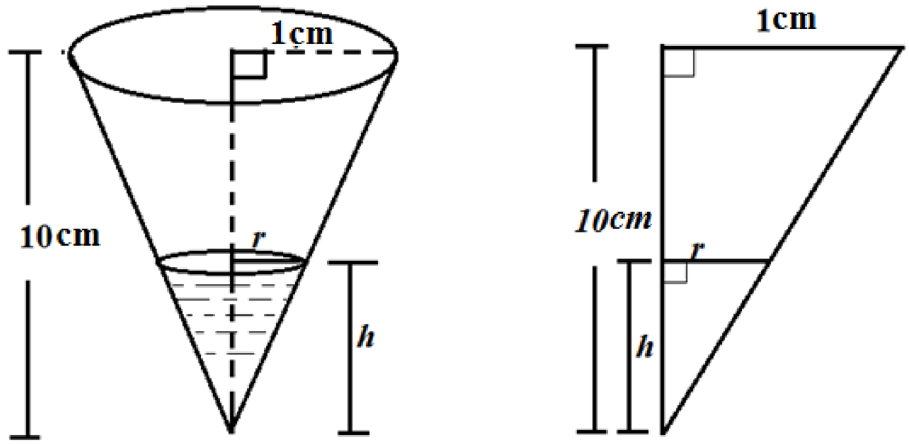
$\frac{dv}{dt}$ **=0.1 and what is required is** $\frac{dA}{dt}$ **at the end they are mentioning when it is ½ way up therefore our variable to use must be terms of height(h)**

From chain rule

$$\frac{dv}{dt} = \frac{dv}{dt} \cdot \frac{du}{dt} \quad and \quad \frac{dA}{dt} = \frac{dA}{dh} \cdot \frac{dh}{dt}$$

From o'level volume of a cone is given by expression $V = \frac{1}{3}\pi r^2 h$

We have two variables r and h so there is need to change r in terms of h



Comparing similar sides of the triangle base and height

$$\frac{r}{1} = \frac{h}{10}$$

$$r = \frac{1}{10}h$$

Substituting for r in expression for volume

$$V = \frac{1}{3}\pi\left(\frac{1}{10}h\right)^2 h = \frac{1}{300}\pi h^3$$

$$\frac{dV}{dh} = \frac{3}{300}\pi h^2 = \frac{\pi h^2}{300}$$

When the cone is half way up h =5cm

$$\frac{dV}{dh} = \frac{\pi(5)^2}{100} = \frac{1}{4}\pi$$

$$from\ \frac{dV}{dt} = \frac{dV}{dh}.\frac{du}{dt}$$

$$0.1 = \frac{1}{4}\pi.\frac{dV}{dt}$$

$$\frac{0.4}{\pi}=\frac{dh}{dt}$$

The surface of the water in the cone is circular therefore formula for area of a circle

$$A=\pi r^2$$

$$A=\pi\left(\frac{1}{10}h\right)^2=\frac{\pi}{100}h^2$$

$$\frac{dA}{dh}=\frac{2\pi h}{100}=\frac{\pi h}{50}$$

But h =5

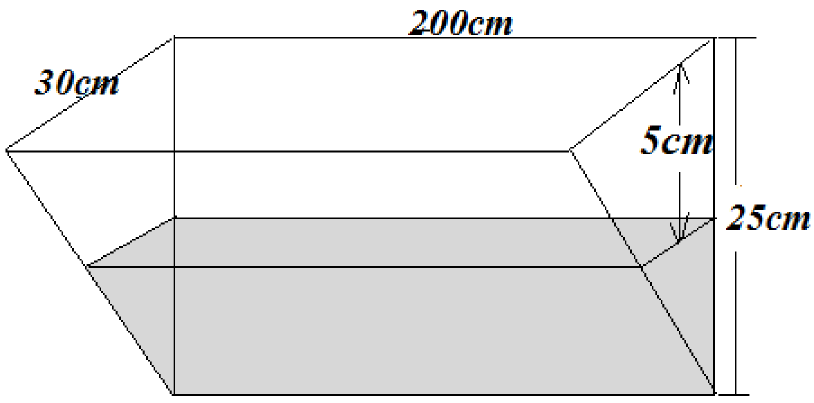
$$\frac{dA}{dh}=\frac{5\pi}{50}=\frac{\pi}{10}$$

$$\frac{dA}{dt}=\frac{dA}{dh}.\frac{dh}{dt}=\frac{\pi}{10}.\frac{0.4}{\pi}=0.04cm^2s^{-1}$$

Example

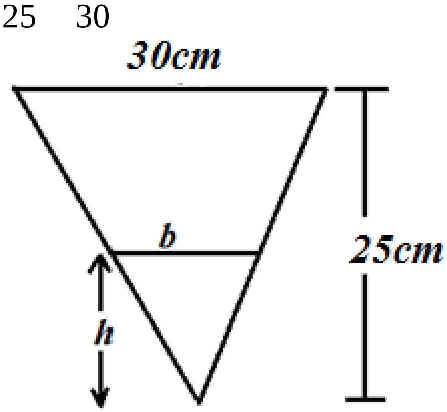
A horse trough has a triangular cross section of height 25cm, base 30cm and 2m long . A horse is drinking steadily and when water is 5cm below the top it is being lowered at a rate of 1cm per Minute . Find the rate of consumption in litres per minute

According to the question the variable considered here is $\frac{dh}{dt}=1$



Comparing similar side

$$\frac{h}{25} = \frac{b}{30}$$



$$\frac{30}{25}h = b$$

$$\frac{6}{5}h = b$$

Volume of a trough = Area of cross section x distance in between

$$V = \frac{1}{2}bh \times 200 = 100bh$$

But $b = \frac{6}{5}h$

$$V = 120h^2$$

$$\frac{dv}{dh} = 240h$$

When water is 5cm blow them the height =20cm

$$\frac{dV}{dh} = \frac{500}{300} \times 20 = \frac{10000}{3} = 240 \times 20 = 4800$$

From chain rule

$$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$$

$= 4800 \ \text{x} \ 1$

$= 4800 \ cm^3 Min^{-1}$

But $1000cm^3 = 1$litre

$$\frac{dV}{dt} = 4.8l \min^{-1}$$

Product And Quotients Rule

Products

If y = uV   where u and V are functions of x

$$\frac{dy}{dx} = u\frac{dV}{dx} + V\frac{du}{dx}$$

Example

Differentiate $y = (x^2 + 1)^2(x+2)^3$

let $u = (x^2+1)^2$

$$\frac{du}{dx} = 2(x^2 + 1)2x = 4x(x^2 + 1)$$

Let $V = (x+2)^3$

$$\frac{dv}{dx} = 3(x^2 + 1)^2$$

$$\frac{dy}{dx} = (x^2+1)^2.3(x^2+2)^2 + (x+2)^3.4(x^2+1)$$

$$= (x^2+1)(x+2)^2\left[3(x^2+1)+4x(x+2)\right]$$

$$= (x^2+1)(x+2)^2\left[3x^2+3+4x^2+8x)\right]$$

$$= (x^2+1)(x+2)^2(7x^2+8x+3)$$

Ensure to simplify up to the end

Example

$$y = (x^2-1)\sqrt{x+1}$$

Let $u = x^2 - 1 \qquad \frac{du}{dx} = 2x$

$$V = (x-1)^{1/2}$$

$$\frac{dV}{dx} = \frac{1}{2}(x-1)^{1/2}(1) = \frac{1}{2\sqrt{x+1}}$$

$$\frac{dy}{dx} = (x^2-1)\frac{1}{2}\left(\frac{1}{\sqrt{x+1}}\right) + (x+1)^{1/2}(2x)$$

$$\frac{dy}{dx} = \left(\frac{(x^2-1)}{2\sqrt{x+1}}\right) + 2x\sqrt{(x+1)}$$

$$\frac{dy}{dx} = \frac{(x^2-1)}{2\sqrt{x+1}} + 2x\sqrt{(x+1)}$$

$$= \frac{x^2-1+4x^2+4x}{2\sqrt{x+1}}$$

$$=\frac{3x^2+4x-1}{2\sqrt{x+1}}$$

Quotient

Given $y=\frac{u}{V}$

$$\frac{dy}{dx}=\frac{V\frac{du}{dx}-u\frac{dv}{dx}}{V^2}$$

Example

Differentiate

$$y=\frac{(x-3)^2}{(x+2)^2}$$

$$Let\ \ u=(x-3)^2,\frac{du}{dx}=2(x-3)$$

$$Let\ \ V=(x+2)^2,\frac{dv}{dx}=2(x+2)$$

$$=\frac{(x+2)^2.2(x-3)-(x+2)^2.2(x+2)}{(x+2)^4}$$

$$=\frac{2(x-3)(5)}{(x+2)^3}$$

$$=\frac{10(x-3)}{(x+2)^3}$$

Ensure to simplify up to the end

Example

Differentiate

$$y=\sqrt{\frac{(x-3)^3}{(x+2)}}=\frac{(x-1)^{3/2}}{(x+2)^{1/2}}$$

Let u= $(x+1)^{3/2}$

$$\frac{du}{dx}=\frac{3}{2}(x+1)^{1/2}(1)$$

$$v=(x+2)^{1/2}$$

$$\frac{du}{dx}=\frac{3}{2}(x+1)^{1/2}(1)$$

Substituting in the formula

$$\frac{dy}{dx}=\frac{(x+1)^{1/2}.\frac{3}{2}(x+1)^{1/2}-(x+1)^{3/2}\frac{1}{2}(x+2)^{-1/2}}{(x+2)}$$

Multiplying the numerator and denominator with $(x+2)^{½}(x+1)^{½}$ in order to remove the fraction powers on the numerator

$$\frac{dy}{dx}=\frac{\left((x+1)^{1/2}.\frac{3}{2}(x+1)^{1/2}-(x+1)^{3/2}\frac{1}{2}(x+2)^{-1/2}\right)(x+1)^{1/2}(x+2)^{1/2}}{(x+2)(x+2)^{1/2}(x+1)^{1/2}}$$

$$\frac{dy}{dx}=\frac{\frac{3}{2}(x+2)(x+1)-(x+2)^2\frac{1}{2}(x+2)^0}{(x+2)^{3/2}(x+1)^{1/2}}$$

$$\frac{dy}{dx}=\frac{\frac{1}{2}(x+1)[3(x+2)-(x+1)]}{(x+1)^{1/2}(x+2)^{3/2}}$$

$$\frac{dy}{dx}=\frac{\sqrt{(x+1)}(2x+5)}{2\sqrt{(x+2)}}$$

$$or\,\frac{dy}{dx}=\sqrt{\frac{(x+1)}{(x+2)^3}}\left(\frac{2x+2}{2}\right)$$

Implicit Functions

These are functions where two variables are mixed up

Example

Differentiate

$x^2+2xy-2y^2+x=2$

$$\frac{d}{dx}(x^2+2xy-2y^2+x-2)$$

$$\Rightarrow(2x+2x\frac{dy}{dx}+2y.1-4y\frac{dy}{dx}+1-0=0$$

$$\Rightarrow(2x+2y+1)-(4y-2x)\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{2x+2y+1}{2(2y-x)}$$

Example

Find $\frac{dy}{dx}$ of the function

$X^2+y^2-6xy+3x-2y+5=0$

$$2x+2y\frac{dy}{dx}-6x\frac{dy}{dx}-6y.1+3-2\frac{dy}{dx}+0=0$$

$$(2x-6y+3)-(6x+2-2y)\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{2x-6y+3}{2(3x+1-y)}$$

Parametric Equations

If both x and y are given out in a different variable

Say x is in terms of t and y is in terms of t

Example

Find the gradient of the curve

$$x=\frac{2t}{t+2} \quad and \quad y=\frac{3t}{t+3}$$

$$\frac{dx}{dt}=\frac{(t+2)(2)-(2t)(1)}{(t+3)^2}=\frac{-3t+9-3t}{(t+3)^2}=\frac{9}{(t+3)^2}$$

$$\frac{dx}{dt}=\frac{dx}{dt}\cdot\frac{dt}{dx}=\frac{9}{(t+3)^2}\cdot\frac{(t+2)^2}{4}=\frac{9(t+2)^2}{4(t+3)^2}$$

Differentiating parametric equations

Given that $x=\frac{t^2}{1+t^3}$, $\quad y=\frac{t^3}{1+t^3}$, find $\frac{dy}{dx}$.

$$\frac{dx}{dt}=\frac{(1+t^3)(2t)-t(3t^3)}{\left(1+t^3\right)^2}$$

$$= t\frac{(2+2t^3-3t^3)}{(1+2t^3)^2}$$

$$= \frac{t(2-t^3)}{(1+t3^3)^2}$$

$$\frac{dy}{dx} = \frac{(1+t^3)(3t^2)-t^3(3t^3)}{(1+t^3)^2}$$

$$= \frac{3t^3+3t^5-3t^5}{(1+t^3)^2}$$

$$= \frac{3t^2}{(1+t^3)^2}$$

$$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = \frac{3t^2}{(1+t^3)^2} \text{ x } = \frac{(1+t^3)^2}{t(2-t^3)} = \frac{3t}{2-t^3}$$

Small changes

From above we already seen that $\frac{\Delta y}{\Delta x} \approx \frac{dy}{dx}$ and Δx tends to zero

$$\therefore \Delta y \approx \frac{dy}{dx}.\Delta x$$

Example

This side of a square is 5cm. Find the increase in the area of the square when the side expands 0.01cm

$A = x^2$

$$\frac{dA}{dx} = 2x$$

When x =5

$$\frac{dA}{dx} = 2 \times 5 = 10$$

And $\Delta x = 0.01$

$$\frac{\Delta A}{\Delta x} \approx \frac{dA}{dx}$$

$$\Delta A = \frac{dA}{\Delta x}.\Delta x$$

$$\Delta A \approx 10 \times 0.01 = 0.1$$

$$\therefore Increase\,in\;Area = 0.1$$

Example

A 2% error is made in measuring the radius of a sphere. Find the percentage error in surface area.

$$S = 4\pi r^2$$

$$\frac{ds}{dr} = 8\pi r$$

$$\frac{\Delta s}{\Delta r} = \frac{ds}{dr}$$

$$\Delta s = \frac{ds}{dr}.\Delta r$$

$$\Delta s = (8\pi r)\Delta r$$

And $\Delta r = {}^{2}\!/_{100}\, r = 0.02r$

$$\Delta s = (8\pi r)(0.2r)$$

$\Delta s \approx 0.16\pi r^2$

$\frac{\Delta s}{s} \times 100$ *is percentage error*

$\frac{0.16\pi r^2}{4\pi r^2} \times 100 = 4\%$

Example

Find the approximation for

$\sqrt{9.01}$

Let $\sqrt{x}$ where x=9

$\frac{dy}{dx} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

When x=9

$\frac{dy}{dx} = \frac{1}{6}$

$But \ \frac{\Delta y}{\Delta x} \approx \frac{dy}{dx}$

$But \ \Delta y \approx \frac{dy}{dx}.\Delta x = \frac{1}{6} \times 0.01 = \frac{0.01}{6}$

$y + \Delta y = \sqrt{9} + \frac{0.01}{6}$

=3.00167

Example

Using small changes

Show that $(244)^{1/5} = 3\frac{1}{405}$

Let $y = x^{1/5}$ $\qquad x = 243$

$\Delta x = 1$

$$\frac{dy}{dx} = \frac{1}{5}x^{-4/5} = \frac{1}{5x^{-4/5}}$$

$$= \frac{1}{5(243)^{4/5}} = \frac{1}{5(3^5)^{4/5}} = \frac{1}{5(3^4)}$$

$$= \frac{1}{5 \times 81}$$

$$= \frac{1}{405}$$

$$But \ \frac{\Delta y}{\Delta x} \approx \frac{dy}{dx}$$

$$But \ \Delta y \approx \frac{dy}{dx}.\Delta x$$

$$\Delta y = \left(\frac{0.01}{6}\right)(1)$$

$$= \frac{1}{405}$$

$$\mathrm{y} + \Delta \mathrm{y} = (243)^{1/5} + \frac{1}{405}$$
$$= (3^5)^{1/5} + \frac{1}{405}$$

$$=3\frac{1}{405}$$

Second Derivative

$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)$ differentiating twice even $\frac{dv}{dt}=\frac{dv}{ds}.\frac{ds}{dt}=V\frac{dv}{ds}$

Also if x and y are in different variable say t

$$\frac{d^2y}{dx^2}=\left(\frac{d}{dx}\left(\frac{dy}{dt}\right)\right).\frac{dt}{dx}$$

Example

Given $y=4x^3-6x^2-9x+1$. Find

$$\frac{dy}{dx} and \frac{d^2y}{dx^2}$$

$$\frac{dy}{dx}=4(x^2)-6(2x^1)-9(1x^{1-1}) \;\; to \;\; \frac{dy}{dx}=12x^2-12x-9$$

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{dy}{dx}(12x^2 12x-9=24x-12$$

Example

If $x=a(t^2-1)$, $y=2a(t+1)$ find $\frac{d^2y}{dx^2}$

$$\frac{dx}{dt}=a(2t)=2at, \frac{dy}{dt}=2a(1)$$

$$\frac{dy}{dx}=\frac{dy}{dt}.\frac{dt}{dx}=\frac{29}{2at}=\frac{1}{t}=t^{-1}$$

$$\frac{d^2y}{dx^2} = \left(\frac{d}{dt}\left(\frac{dy}{dx}\right)\right)\frac{dt}{dx}$$

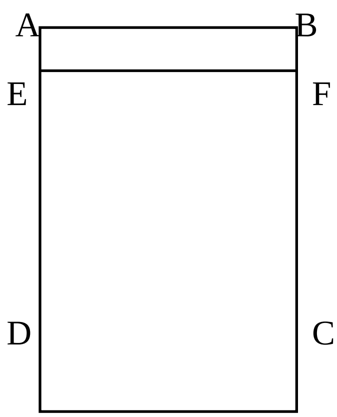
$$= \left(\frac{d}{dt}\left(t^{-1}\right)\right)\frac{1}{2at}$$

$$= \frac{-1}{t^2}.\frac{1}{2at}$$

$$= \frac{-1}{2at^3}$$

Exercise 1

1. Find the derivative of f(x) $= \dfrac{x^4+3x^2}{2x^2}$
2. Find the derivative of $f(x) = (x^2+2)(x-4)$
3. Find the equation of the tangent at point P(3,9) to the curve y= $x^3+6x^2+15x-9$ .If O is the origin and N is the foot of the perpendicular from P to the x-axis . Prove that the tangent at P passes through the mid point of ON . Find the coordinates of another point on the curve , the tangent at which is parallel to the tangent at the point (3,9)
4. The figure below represents the end view of the outer cover of a match box AB and EF being C gummed together and assumed to be of the same length. If the total length of the edge (ABCDEF) is 12cm . Calculate the lengths of AB and BC which will give the maximum possible area .



5. Sketch the curve $y = 4x^3 - 3x^4$ Showing clearly the turning points and points where the curve crosses the axes.
6. Differentiate with respect to x

$$y = (1 - x^2)(1 - 2x)^{1/3}$$

7. Differentiate

$$y = \sqrt{\frac{(x+2)^3}{(x-1)}}$$

8. Find $\frac{dy}{dx}$ of $x^2 - 3xy + y^2 - 2y + 4x = 0$
9. Find $\frac{dy}{dx}$, Given $x = \frac{t}{1-t}$ *and* $y = \frac{1-2t}{1-t}$
10. Find the approximation of Find $\sqrt[3]{65}$
11. Given $y = \frac{x^2}{\sqrt{x+1}}$ find $\frac{d^2y}{dx^2}$
12. If $x = (t^2-1)^2$ and $y = t^3$ find $\frac{d^2y}{dx^2}$

TOPIC 9: DIFFERENTIAL EQUATIONS

The differential equation defines a family of curves

A general solution involves one or more arbitrary constant is the equation of any member of the family

A particular solution is the equation of one member of the family

The order of a differential equation is determined by the highest differential coefficient present .

Note that the syllabus only allows us to cover first order

1. First order separating the variables

$$x^2\frac{dy}{dx}=y(y-1)$$

Note that if x and y can be separated

$$\frac{dy}{y(y-1)}=\frac{1}{x^2}dx$$

$$\frac{1}{y(y-1)}=\frac{A}{y}+\frac{B}{y-1}$$

$$1\equiv A(y-1)+By$$

For y =1   y =0

1=B   1=-A

A=-1

$$-\int\frac{dy}{y}+\int\frac{dy}{y-1}=\int x^{-2}dx$$

$$-Iny+In(y-1)=\frac{-1}{x}+C$$

Example

$$x\frac{dy}{dx}-3=2\left(y+\frac{dy}{dx}\right)$$

$$x\frac{dy}{dx}-3=2y+2\frac{dy}{dx}$$

$$(x-2)\frac{dy}{dx}=(2y+3)$$

$$\frac{dy}{2y+3}=\frac{1}{x-2x}dx$$

$$\frac{1}{2}\int\frac{2dy}{2y+3}=\int\frac{dx}{x-2}$$

$$\frac{1}{2}In(2y+3)=In(x-2)+InA$$

$$In\sqrt{2y+3}=In(x-2)$$

$$\sqrt{2y+3}=A(x-2)$$

First Order Exact Equation

An exact equation is one with one side originates from some derivative

Example

$$x^2\frac{dy}{dx}+2xy=1$$

Taking the L.H.S

$$\frac{d}{dx}\left(x^2.y\right)=x^2\frac{dy}{dx}+y.2x$$

Since they are equal with above expression it is taken to be exact

So $\frac{d}{dx}\left(x^2.y\right)=1$

$$\int\frac{d}{dx}\left(x^2y\right)dx=\int 1dx$$

$$x^2y=x+C$$

Example

$$x^2Cosu\frac{du}{dx}+2xSinu=\frac{1}{x}$$

$$\frac{d}{dx}\left(x^2.Sinu\right)=x^2Cosu\frac{du}{dx}+\left(Sinu\right).2x$$

$$\therefore\frac{d}{dx}\left(x^2Sinu\right)=\frac{1}{x}$$

$$\int\frac{d}{dx}\left(x^2Sinu\right)dx=\int\frac{1}{x}dx$$

x^2Sinu=Inx+c

Integrating Factor

Differential equations which are not exact can be made exact if you use an integrating factor. This is only possible if the equation is not exact and can be written in this form

$\frac{dy}{dx}+Py=Q$ where P and Q are function in terms of what your differentiating with respect to

Example

$$\frac{dy}{dx}-y\tan x=x$$

$$I.F=e^{\int\tan x dx}$$

$$=e^{\int\frac{-\sin x}{\cos x}dx}=e^{InCosx}=Cosx$$

$$\therefore\cos x\frac{dy}{dx}-y\tan x\cos x=x\cos x$$

$$\frac{dy}{dx}(y\cos x)=x\cos x$$

$$\int\frac{d}{dx}(y\cos x)dx=\int x\cos x dx$$

ycosx=xsinx-$\int\sin x dx$

Let u=A $\qquad \frac{du}{dx}=1$

$$Cosx = \frac{dv}{dx}$$

V =Sinx

ycosx =xsinx+cosx+C

Example

$$x\frac{dy}{dx} - y = \frac{x}{x-1}$$

$$\frac{dy}{dx} - \frac{1}{x}y = \frac{1}{x-1}$$

$$I.F = e^{\int \frac{-1}{x}dx} = e^{\int \frac{-1}{x}dx} = e^{-Inx} = e^{-Inx} = \frac{1}{x}$$

$$\frac{1}{x}\frac{dy}{dx} - \frac{1}{x^2}y = \frac{1}{x(x-1)}$$

$$\frac{d}{dx}\left(\frac{y}{x}\right) = \frac{1}{x(x-1)}$$

$$\int \frac{d}{dx}\left(\frac{y}{x}\right)dx = \int \frac{dx}{x(x-1)}$$

$$\frac{y}{x} = \int \frac{dx}{x(x-1)}$$

$$\frac{1}{x(x-1)} \equiv \frac{A}{x} + \frac{B}{x-1}$$

$$1 \equiv A(x-1) + Bx$$

For x=1 for x=0

1=B 1=-A

A=-1

$$\frac{1}{x(x-1)} \equiv \frac{1}{x-1} - \frac{1}{x}$$

$$\int \frac{dx}{x(x-1)} = \int \frac{dx}{x-1} - \int \frac{dx}{x}$$

=In(x-1)-Inx +C

$$= In\left(\frac{x-1}{x}\right) + C$$

$$\Rightarrow \frac{y}{x} In\left(\frac{x-1}{x}\right) + C$$

Example

$$(x-2)\frac{dy}{dx} + 3y = \frac{2}{x-2}$$

$$\frac{dy}{dx} + \left(\frac{3}{x-2}\right)y = \frac{2}{(x-2)^2}$$

$$= I.F = e^{\int \frac{3}{x-2}dx} = e^{3\int \frac{dx}{x-2}} = e^{3In(x-2)} = e^{In(x-2)^3} = (x-2)^3$$

$$(x-2)^3 \frac{dy}{dx} + 3(x-2)^2 y = 2(x-2)$$

$$\frac{d}{dx}\left(y(x-2)^3\right) = \int 2x - 4dx$$

$$y(x-2)^3 = x^2 - 4x + C$$

First Order Homogeneous Equations

In this part all the terms are taken to have same dimensions

Example

$$xy\frac{dy}{dx} = x^2 + y^2$$

Dividing through by x^2

$$\frac{xy}{x^2}\frac{dy}{dx} = 1 + \frac{y^2}{x^2}$$

$$\left(\frac{y}{x}\right)\frac{dy}{dx}=1+\left(\frac{y}{x}\right)^2$$

Let u = $\frac{y}{x}$

y = ux

$$\frac{dy}{dx}=u+x\frac{du}{dx}$$

$$u\left(u+x\frac{du}{dx}\right)=u+u^2$$

$$u^2+ux\frac{du}{dx}=1+u^2$$

$$ux\frac{du}{dx}=1$$

$$\int udu=\int\frac{1}{x}dx$$

$\tan^{-1}$u=Inx +InA

Tan^{-1}u= InAx

U= tan(InAx0

$$\frac{y}{x}=\tan(InAx)$$

$$y=x\tan(InAx)$$

EXAMPLE

$$\frac{du}{d\theta}+uCot\theta=2Cos\theta \quad Given \quad u=3 \quad when \quad \theta=\frac{\pi}{2}$$

$$I.F=e^{\int Cot\theta d\theta}=e^{\int\frac{Cot\theta d\theta}{Sin\theta}}=e^{\int In\sin\theta}=Sin\theta$$

$$Sin\theta\frac{du}{d\theta}+u\cot\theta\sin\theta=2\sin\theta\cos\theta$$

$$\frac{d}{d\theta}(uSin\theta)=\sin 2\theta$$

$$\int \frac{d}{d\theta}(uSin\theta)d\theta = \int \sin 2\theta d\theta$$

$$uSin\theta = -\frac{1}{2}Cos2\theta + C$$

$$When \ \ \theta = \frac{\pi}{2} \ \ , \ u = 3$$

$$3\sin\frac{\pi}{2} = \frac{1}{2}Cos \ \pi + C$$

$$U\sin\theta = -\frac{1}{2}Cos \ \theta + \frac{5}{2} \text{ Particular solution}$$

Exercise

1. $x\dfrac{dy}{dx} = y + xy$
2. $2\sin\theta\dfrac{d\theta}{dr} = Cos\theta - Sin\theta$
3. $e^x\dfrac{du}{dx} = y^2 + 4 = 0$
4. $r\sec^2\theta + 2\tan\theta\dfrac{d\theta}{dr} = \dfrac{2}{r}$
5. $x^2\dfrac{dy}{dx} = 3x^2 + xy$
6. $x\dfrac{dy}{dx} = y\sqrt{x^2 + y^2}$
7. $\dfrac{dy}{dx} = x\dfrac{(x - y + 2)}{x + y}$
8. $\dfrac{dy}{dx} = \dfrac{2x + y - 2}{2x + y + 1}$

Formation Of Differential Equatorial

In this category of question you are supposed to form a different equation using the information given

There are some key words to note for example rate , increasing or decreasing, growth decay. In some cases they use gradient, velocity , acceleration etc

A learner should note what is changing and with respect to what also not whether that leads to increase or decrease of the variable if it increases the rate will be positive and negative when it decrease.

According to Newton's law of cooling the rate at which the temperature of a body falls is proportional to the amount by which the temperature exceeds that of its surrounding temperature .

Suppose the temperature of the object falls from 200^0c to 100^0c in 40 minutes in a surrounding of 10^0c . Prove that after t minutes the body is given by $T=10+190e^{-kt}$ where $k=\frac{1}{40}In\left(\frac{19}{9}\right)$

Calculate the time it takes to reach 50^0c

$$\frac{dT}{dt}\alpha\left(T-10^0\right)$$

$$-\frac{dT}{dt}=K\left(T-10^0\right)$$

$$\frac{dT}{T-10}=-kdt$$

$$\int\frac{dT}{T-10}=\int-kdt$$

In(T-10) = -Kt+C

When T = 200, t=0

T =100 , t=40

In(200-10) = 0+C

In190=C

In(T-10)= -Kt + In190

$$In\left(\frac{T-10}{190}\right)=-kt$$

In(100-10) =-40k+In190

In(90)-In(190) =-40K

$$In\left(\frac{90}{190}\right) = -40K$$

$$In\left(\frac{9}{19}\right) = -40K$$

$$-\frac{1}{40}In\left(\frac{19}{9}\right) = K$$

$$\frac{1}{40}In\left(\frac{19}{9}\right) = K$$

$$In\left(\frac{T-10}{190}\right) = -kt$$

$$e^{In\left(\frac{T-10}{190}\right)} = e^{-kt}$$

$$\frac{T-10}{190} = e^{-kt}$$

T-10=190e^{-kt}

T= 10+190e^{-Kt}

$$In\left(\frac{T-10}{190}\right) = -kt$$

$$In\left(\frac{50-10}{190}\right) = -\frac{1}{40}In\left(\frac{19}{9}\right)t$$

$$\frac{In\left(\frac{4}{19}\right)}{\frac{1}{40}In\left(\frac{19}{9}\right)} = -t$$

T=83.4 Minutes

Note

These are the common questions set of formation of differentiation equation

1. Newton's law of cooling

   Relating change in Temperature and the excess temperature due temperature of the body and the surrounding T is temperature at any time and A Temperature of the surrounding.

   $$\frac{dT}{dt}\alpha(T-A)$$

   $$\frac{dT}{dt}=-K(T-A) \text{ Or } \frac{dT}{dt}=K(T-A)$$

   K is negative when cooling takes place and positive when temperature increases

2. **Falling bodies**

   If the body falls from rest in a medium which causes the velocity to decrease at a rate proportional to the velocity then $-\frac{dv}{dt}\alpha V$

   $$\frac{dv}{dt}=-KV$$

   But if it increases as it falls

   $$\frac{dv}{dt}\alpha V \qquad \text{and } \frac{dv}{dt}=KV$$

3. Growth of yeast cells in a culture it the number of cells are n at any time t. If the rate at which the number increases is proportion to the number of cells. $\frac{dn}{dt}\alpha\, n$

   $\frac{dn}{dt}=Kn$ Since it grows or increases rate will be positive.

4. A chemical mixture contains two substances A and B whose weight, are W_A and W_B and whose combined weight remains constant B is converted into A at a rate which is inversely proportional to the weight B and proportional to square of A in the mixture at any time ***t***. The weight B present at time t can be found using

   $\frac{dw_B}{dt}\alpha\frac{{w_A}^2}{W_B}$ Since its B which is changing to A therefore it is the variable and increasing grad function will be positive

$$\frac{dw_B}{dt} = \frac{Kw_A{}^2}{W_B}$$

But $W=W_A + W_B$

Where W is a constant

$W_A=W-\mathbf{W}_B$

$$\frac{dw_B}{dt} \alpha \frac{K(w-w_B)^2}{W_B}$$

Example

A substance loses mass at a rate which is proportional to the amount M present at a time t

(a) For a differential equation connecting m, t and a constant of proportionality K

(b) If initially the mass of the substance is M_0 show that $M=M_oe^{-Kt}$

(c) Given that half of the substance is lost in 1600 years. Determine the number of years 15g of the substance would reduce to 13.6g

(d) Since it is loosing mass

$$-\frac{dm}{dt} \alpha\ m$$

$$\frac{dm}{dt} = -Km$$

$$\frac{dm}{dm} = -Kdt$$

$$\int \frac{dm}{dm} = \int -Kdt$$

Inm =-Kt +C

$t=0$, $m = m_o$

$Inm_o=0+C$

$C = Inm_o$

$Inm-Inm_o=-Kt$

$$In\left(\frac{m}{m_o}\right) = -Kt$$

$$\frac{m}{m_o} = e^{-Kt}$$

$$m = m_o e^{-Kt}$$

$t = 1600$, $m = \frac{1}{2} m_o$

$$In\left(\frac{\frac{1}{2}m_0}{m_0}\right) = -1600k$$

$$In\frac{1}{2} = -1600k$$

$In1 - In2 = -1600K$

$-In2 = -1600k$

$$\frac{1}{1600}In2 = K$$

$M_o = 15g$

$M = 13.6g$

$$In\left(\frac{13.6}{15}\right) = \frac{In2}{-1600}t$$

$$\frac{-1600In\left(\frac{13.6}{15}\right)}{In2} = t$$

$t = 4years$

Example

A bacteria in a culture increase at a rate proportional to the number of bacterial present .

If the number increases from 1000 to 2000 in one hour.

(a) How may bacterial will be present after 1 ½ hours

(b) How long will it take for the number of bacteria in the culture to become 4000

Let the number of bacterial present be P.

$$\frac{dp}{dt} \alpha P$$

$$\frac{dp}{dt} = KP$$

$$\frac{dp}{p} = Kdt$$

$$\int \frac{dp}{p} = \int Kdt$$

InP =Kt +C

t= 0, P=1000

t=1 P=2000

In(1000)=0+C

C=In1000

InP-In1000=Kt

$$In\left(\frac{p}{1000}\right) = Kt$$

$$In\left(\frac{2000}{1000}\right) = K$$

In2= K

$$In\left(\frac{p}{1000}\right) = Kt$$

$$e^{In\left(\frac{p}{1000}\right)} = e^{Kt}$$

$$\frac{p}{1000} = e^{Kt}$$

$$p = 1000 e^{(In2)t}$$

$$p = 1000 e^{(In2)\left(\frac{3}{2}\right)}$$

$$In\left(\frac{4000}{1000}\right) = (In2)t$$

$$\frac{In(4)}{(In(2)} = t$$

Example

The acceleration of a particle after t seconds is given by a =5+Cos ½t . If initially the particle is moving at 1ms^{-1}. Find the velocity after $2\pi\, Seconds$ and the distance it had covered by then.

$$a = \frac{dv}{dt}$$

$$\frac{dv}{dt} = (5 + Cos\frac{1}{2}t)$$

$$\int dv = \int 5 + Cos\frac{1}{2}tdt$$

$$v = 5t + 2Sin\frac{1}{2}t + C$$

t= 0 , V=1

1=0+C

C=1

$$v = 5t + 2\sin\frac{1}{2}t + 1$$

$$v = 5(2\pi) + 2\sin\pi + 1$$

$$v = (10\pi + 1)ms^{-1}$$

$$v = \frac{dx}{dt}$$

$$\frac{dx}{dt} = 5t + 2\sin\frac{1}{2}t + 1$$

$$dx = \left(5t + 2\sin\frac{1}{2}t + 1\right)dt$$

$$\int dx = \int\left(5t + 2\sin\frac{1}{2}t + 1\right)dt$$

$$x = \frac{5}{2}t^2 - 4Cos\frac{1}{2}t + t + C$$

From above

t =0 , x=0

0=0-4+0+C

C=4

$$x = \frac{5}{2}(2\pi)^2 - 4Cos\pi + 2\pi + 4$$

x= (112.98)m

Example

A rumour spreads through a town at a rate which is proportional to the product of the number of people who have heard the rumour and those who have not heard it. Given that x is a fraction of those who have heard the rumour at any time *t*

(i) Form a differential equation connecting *x, t* and constant *k*

(ii) If initially a fraction C of the population had heard the rumour deduce that

$$x = \frac{C}{C + (1 - C)e^{-Kt}}$$

(iii) Given 15% had heard the rumour at 9.00 a.m and another 15% by noon . Find what further fraction of the population would have heard the rumour by 3.00P.m

The population who have not heard the romour =1-x

$$\frac{dx}{dt}\alpha x(1 - x)$$

$$\frac{dx}{dt} = Kx(1 - x)$$

$$\frac{dx}{x(1 - x)} = Kdt$$

$$\int \frac{dx}{x(1-x)} = \int K dt$$

$$\frac{1}{x(1-x)} \equiv \frac{A}{x} + \frac{B}{(1-x)}$$

$$1 \equiv A(1-x) + Bx$$

For x=0

1=A

When x=1

1=B

$$\Rightarrow \frac{1}{x} + \frac{1}{1-x}$$

$$\int \frac{dx}{x(1-x)} = \int \frac{dx}{x} + \int \frac{dx}{1-x}$$

$$\therefore In x - In(1-x) = Kt + D$$

$$In\left(\frac{1}{1-x}\right) = Kt + D$$

t = 0 x=c

$$In\left(\frac{C}{1-C}\right) = 0 + D$$

$$D = In\left(\frac{C}{1-C}\right) =$$

$$In\left(\frac{x}{1-x}\right) = Kt + In\left(\frac{C}{1-C}\right)$$

$$In\left(\frac{x}{1-x}\right) - In\left(\frac{C}{1-C}\right) = Kt$$

$$let\ \frac{C}{1-C} = A$$

$$In\left(\frac{x}{1-x}\right) - InA = Kt$$

$$In\frac{x}{A(1-x)}=Kt$$

$$e^{In\frac{x}{A(1-x)}}=e^{Kt}$$

$$\frac{x}{A(1-x)}=e^{Kt}$$

$$x=InA(1-x)e^{Kt}$$

$$x=Ae^{kt}-Axe^{Kt}$$

$$x+Axe^{Kt}=Ae^{kt}$$

$$x(1+Ae^{Kt})=Ae^{kt}$$

$$x=\frac{Ae^{kt}}{1+Ae^{Kt}}=\frac{\left(\frac{C}{1-C}\right)e^{kt}}{1+\left(\frac{C}{1-C}\right)e^{Kt}}$$

$$x=\frac{Ce^{kt}}{(1-C)+Ce^{kt}}$$

$$x=\frac{Ce^{kt}.Ce^{-kt}}{(1-C)Ce^{-kt}+Ce^{kt}Ce^{-kt}}$$

$$x=\frac{C}{C+(1-C)e^{-kt}}$$

C=15% t =0 time 9:00am , x = 30%, t =3hrs

$$x=\frac{\frac{15}{100}}{\frac{15}{100}+\left(\frac{85}{100}\right)e^{-3k}}$$

$$\frac{30}{100}=\frac{\frac{15}{100}}{\frac{15}{100}+\left(\frac{85}{100}\right)e^{-3k}}$$

$$\frac{30}{100}=\frac{15}{15+85e^{-3k}}$$

$$\frac{3}{10}=\frac{3}{3+17e^{-3k}}$$

$$3+17e^{-3K}=10$$

$$17e^{-3K}=7$$

$$e^{-3K}=\frac{7}{17}$$

t = 6

$$x=\frac{\frac{15}{100}}{\frac{15}{100}+\left(\frac{85}{100}\right)e^{-6K}}$$

$$x=\frac{15}{15+85e^{2(-3K)}}=\frac{3}{3+17e^{(-3K)^2}}$$

$$x=\frac{3}{3+17\left(\frac{7}{17}\right)^2}$$

$$x=\frac{3}{3+\frac{17\times 49}{17}}$$

$$x=\frac{3\times 17}{3\times 17+49}$$

$$x=\frac{52}{52+49}=\frac{52}{101}=0.5148\approx 51.4\%$$

Further fraction $=51.4\%-15\%$

$=36\%$

Exercise

1. The rate of change of atmospheric pressure P with respect to altitude h in km is proportional to the pressure P if the pressure P_0 at sea level. Find the formula of the pressure at any height.

2. At3:00pm the temperature of a hot metal was 80^0 and that of the surroundings 20^0C. At 3:03 pm the temperature of the metal had dropped to 42^0C. The rate of cooling of the metal was directly proportional to the difference between its temperature T and that of the surroundings.
   (a) Write down the differential equation to represent the rate of cooling of the metal
   (b) Solve the differential equation
   (c) Find the temperature of the metal at 3:05pm
3. An Athlete runs at a speed proportional to the square root of the distance he still has to cover .If the athlete starts running at $10ms^{-1}$ and has a distance of 1600m to cover . Find how long he will take to cover this distance
4. The differential equation $\frac{dp}{dt} = kP(c-P)$ show a rate at which information flows in a students population C, P represents the number who have heard the information in t days and k is a constant.
   (a) Solve the differential equation
   (b) A school has a population of 1000 students initially 20 students had heard the information . A day later 50 students had heard the information. How many students heard the information by the tenth day.
5. A research investigates the effect of a certain chemicals on a virus infection crops, revealed that the rate at which the virus population is destroyed is directly proportional to the population at the time . Initially the population was P_o at t months later it was found to be P
   (a) Form a differential equation connecting P and t
   (b) Given that the virus population reduced to one third of the initial population in 4 months solve the equation above.

Permutations and combinations

Permutation

A permutation is an ordered arrangement of a number of objects

Consider digits 1, 2 and 3; find the possible arrangements of the digits

$$\left.\begin{matrix} 123, 132, 321 \\ 231, 321, 321 \end{matrix}\right\} \text{The total of six}$$

This problem may also be solved as follows:

Given the three digits above, the first position can take up three digits, the second position can take up two digits and the third position can take up 1 digit only

1st position	2nd position	3rd position
3	2	1

The total is thus 3 x 2 x 1 = 6

If the digits were four say 1, 2, 3, 4 the arrangement would be

1st	2nd	3rd	4th
4	3	2	1

The total is thus 4 x 3 x 2 x 1 = 24

In summary the number of ways of arranging n different items in a row is given by n(n – 1)(n – 2)(n -3) x ……….2 x 1 and can be expressed as n!

If the total number of books is 6

The total number of arrangements = 6!

= 6 x 5 x 4 x 3 x 2 x 1 = 720 ways

Example 1

Find the values of the following expression

(a) 5!
Solution
5!= 5 x 4 x 3 x 2 x 1 = 120

(b) $\frac{8!}{5!}$
Solution
$\frac{8!}{5!} = \frac{8\,x\,7\,x\,6\,x\,5!}{5!} = 336$

(c) $\frac{10!}{6!x\ 5!\ x\ 2!}$
Solution
$\frac{10!}{6!x\ 5!\ x\ 2!} = \frac{10\ x\ 9\ x\ 8\ x\ 7\ x\ 6!}{6!x\ 5\ x\ 4\ x\ 3\ x\ 2\ x\ 1\ x\ 2x\ 1} = 21$

(d) Four different pens and 5 different books are to be arranged on a row. Find
(i) The number of possible arrangements of items
Solution
Total number of items = 4 + 5 = 9
Total number of arrangements = 9!
= 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1
= 362,880 ways
(ii) The number of possible arrangements if three of books must be kept together
Solution
The pens are taken to be one since they are to be kept together. So we consider total number of items to six.
The number of arrangements of six items = 6! = 6 x 5 x 4 x 3 x 2 x 1
= 720 ways
The arrangement of 4 pens = 4!
= 4 x 3 x 2 x 1 =24
Total number arrangements of all the items = 720 x 24 = 17, 280

Multiplication principle of permutation

If one operation can be performed independently in a different ways and the second in b different ways, then either of the two events can be performed in (a + b) ways

Example 2

There are 6 roads joining P to Q and 3 roads joining Q to R. Find how many possible routes are from P to R

From P to Q = 6 ways

From Q to R = 3 ways

Number of routes from P to R = 6 x 3 = 18

Example 3

Peter can eat either matooke, rice or posh on any of the seven days of the week. In how many ways can he arrange his meals in a week

Solution

For each of the 7 days, there are 3 choices

Total number of arrangements

= 3 x 3 x 3 x 3 x 3 x 3 x 3 = 3^7 = 2187 ways

Example 4

There are four routes from Nairobi to Mombasa. In how many different ways can a taxi go from Nairobi to Mombasa and returning if for returning:

(a) any of the route is taken
= 4 x 4 = 16 ways
(b) the same route is taken
= 4 x 1 = 4 ways
(c) the same route is not taken
= 4 x 3 = 12 ways

Example 5

David can arrange a set of items in 5 ways and John can arrange the same set of items in 3 ways. In how many ways can either David or John arrange the items?

Solution

Number of ways in which David arranges = 5

Number of ways in which John arranges = 3

Number of ways in which either David or John arrange the items = 5 + 3 = 8 ways

The number of permutation of r objects taken from n unlike objects

The permutation of n unlike objects taking r at a time is denoted by ${}^nP_r$ which is defined as

${}^nP_r = \frac{n!}{(n-r)!}$, where r ≤ n.

In case r = n, we have ${}^nP_n$ which is interpreted as the number of arranging n chosen objects from n objects denoted by n!

${}^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n! => 0! = 1$

Example 6

How many three letter words can be formed from the sample space {a, b, c, d, e, f}

Solution

Total number of letters = 6 and r = 3

Total number of worms = ${}^6P_3$

$= \frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{6\ x\ 5\ x\ 4\ x\ 3!}{3!} = 120$ways

Example 7

Find the possible number of ways of arranging 3 letters from the word MANGOES

Solution

Total number of letter in the word = 7

and r = 3

Number of ways ${}^7P^3 = \frac{7!}{(7-3)!}$

$=\frac{7\ x\ 6\ x\ 5\ x\ 4\ x\ 3!}{3!} = 840$ways

Example 8

Find number of ways of arranging six boys from a group of 13

Solution

Number of arrangements = $^{13}P_6$

$=\frac{13!}{(13-6)!} = \frac{13!}{7!}$

$= \frac{13x12x11x10\ x9\ x\ 8\ x\ 7!}{7!}$

= 1235520 ways

The number of permutations of n objects of which r are alike

The number of permutations of n objects of which r are alike is given by $\frac{n!}{r!}$

Example 9

Find the number of arranging in a line the letters B, C, C, C, C, C, C

The number of ways of arranging the seven letters of which of which 6 are alike $= \frac{7!}{6!} = \frac{7\ x\ 6!}{6!} = 7$ ways

The number of ways of permutations on n objects of which p of one type are alike, q of the second type are alike, r of the third type are alike, and so on.

The number of ways of permutations on n objects of which p of one type are alike, q of the second type are alike, r of the third type are alike given by $\frac{n!}{p!\ x\ q!\ x\ r!}$

Example 10

Find the possible number of ways of arranging the letter of the word MATHEMATICS in line

Solution

The word MATHEMATICS has 11 letters and contains 2 M, 2A and 2T repeated

The number of ways = $\frac{11!}{2!x\ 2!\ x\ 2!}$

$= \frac{11\ x\ 10\ x\ 9\ \ x\ 8\ x\ 7\ x\ 6\ x\ 5\ x\ 4x3\ x\ 2\ x\ 1}{2\ x\ 2\ x\ 2}$

= 4,989,600

Example 11

Find the possible number of ways of arranging the letter of the word 'MISSISSIPPI' in line

Solution

The word 'MISSISSIPPI' has 10 letters with 4I, 4S, and 2P

The number of ways = $\frac{11!}{4!x\ 4!\ x\ 2!} = 34650$

The number of permutations of the like and unlike objects with restrictions

One should be cautious when handling these problems

Example 12

Find the possible number of ways of arranging

The letters of the word MINIMUM if the arrangement begins with MMM?

Solution

There is only one way of arranging MMM

The remaining contain four letters with 2I can be arranged in

$\frac{1\ x\ 4!}{2!} = \frac{1\ x\ 4\ x\ 3\ x\ 2!}{2!} = 12$ ways

Example 13

(a) How many 4 digit number greater than 6000 can be formed from 4, 5, 6, 7, 8 and 9 if:

(i) Repetitions are allowed
Solution
The first digit can be chosen from 6, 7, 8 and 9, hence 4 possible ways, the 2nd, 3rd and 4th are chosen from any of the six digits since repetitions are allowed

position	1st	2nd	3rd	4th
selections	4	6	6	6

Number of ways
= 4 x 6 x 6 x 6 = 864 ways

(ii) Repetition are not allowed
The first can be chosen from 6, 7, 8 and 9, hence 4 possible ways, the 2nd from 5, 3rd from 4 and 4th from 3 since no repetitions are allowed

position	1st	2nd	3rd	4th
selections	4	5	4	3

Number of ways
= 4 x 5 x 4 x 3 = 240 ways

(b) Find how many four digit numbers can be formed from the six digits 2, 3, 5, 7, 8 and 9 without repeating any digit.
Find also how many of these numbers
(i) Are less than 7000
(ii) Are odd

Solution

position	1^{st}	2^{nd}	3^{rd}	4^{th}
selections	6	5	4	3

Total number of ways = 6 x 5 x 4 x 3 = 360

(i) The 1^{st} number is selected from three (2, 3, 5), the 2^{nd} number from 5, the 3^{rd} from 4 and the 4^{th} from 3 digits

position	1^{st}	2^{nd}	3^{rd}	4^{th}
selections	6	5	4	3

Total number of less than 7000

= 3 x 5 x 4 x 3 = 180

(ii) The last number is selected from four odd digits (3, 5, 7, and 9), the 1^{st} number selected from five remaining, 2^{nd} from 4 and 3^{rd} from 3

position	1^{st}	2^{nd}	3^{rd}	4^{th}
selections	5	4	3	4

Total number of odd numbers formed

= 5 x 4 x 3 x 4 = 240

(c) How many different 6 digit number greater than 500000 can be formed by using the digits 1, 5, 7, 7, 7, 8
Solution
The 1^{st} digit is selected from five (5, 7, 7, 7, 8), the 2^{nd} from remaining five, 3^{rd} from four, 4^{th} from three, 5^{th} from two and 6^{th} from one

1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6th
5	5	4	3	2	1

Total number = $\frac{5\ x\ 5\ x\ 4\ x\ 3\ x\ 2\ x\ 1}{3!}$ = 100
NB. The number is divided by 3! Because 7 appears three times

(d) How many odd numbers greater than 60000 can be formed from 0, 5, 6, 7, 8, 9, if no number contains any digit more than once
Solution
Considering six digits
Taking the first digit to be odd, the first digit is selected from 3 digits (5, 7, 9) and the last is selected from 2 digits

1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6th
3	4	3	2	1	2

Number of ways = 3 x 4 x 3 x 2 x 1 x 2
= 144
Taking the first digit to be even, the first digit is selected from 2 digits (6, 8) since the number should be greater than 60000 and the last is selected from 2 odd digits

1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6th
2	4	3	2	1	3

Number of ways = 3 x 4 x 3 x 2 x 1 x 2
= 144
Considering five digits
Taking the first digit to be odd, the digit greater than 6 are 7 and 9 so first digit is selected from 2 digits and the last is selected from 2 digits

1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
2	4	3	2	2

Number of ways = 2 x 4 x 3 x 2 x 2
= 96
Taking the first digit to be even, the first digit is selected from 2 digits (6, 8) since the number should be greater than 60000 and the last is selected from 3 odd digits (5, 7, 9)

1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
2	4	3	2	3

Number of ways = 2 x 4 x 3 x 2 x 3
= 144
The total number of selections
= 144 + 144 + 96 + 144 = 528

Example 14

The six letter of the word LONDON are each written on a card and the six cards are shuffled and placed in a line. Find the number of possible arrangements if

(a) The middle two cards both have the letter N on them

Solution

If the middle letter are NN, the we need to find the number of different arrangements of the letter LODO. With the 2O's, the number of arrangements = $\frac{4!}{2!} = 12$

(b) The two cards with letter O are not adjacent and the two cards with letter N are also not adjacent

Solution

If the two cards are not adjacent, the number of arrangements = Total number of arrangements of the word LONDON – number of arrangements when the two letters are adjacent

= = $\frac{6!}{2!2!} - 24 = 156$

Example 15

In how many different ways can letters of the word MISCHIEVERS be arranged if the S's cannot be together

Solution

There are 11 letters in the word MISCHIEVERS with 2S's, 2I's and 2E's

Total number of arrangements

= $\frac{11!}{2!2!2!}$ = 4989600

If S's are together, we consider them as one, so the number of arrangements

= $\frac{10!}{2!2!}$ = 907200

∴ the number of possible arrangements of the word MISCHIEVERS when S's are not together

= 4989600 – 907200 = 4082400

The number of permutation of n different objects taken r at a time, if repetition are permitted

Example 16

How many four digit numbers can be formed from the sample space {1, 2, 3, 4, 5} if repetitions are permissible

Solution

The 1st position has five possibilities, the 2nd five, the 3rd five, the 4th five

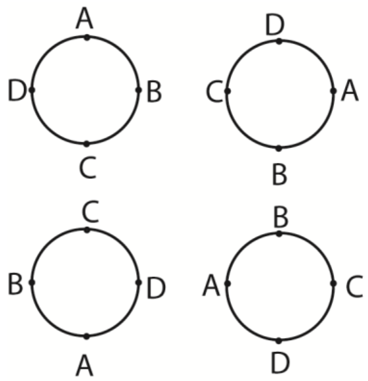
Number of permutations = 5 x 5 x 5 x 5 = 625

Circular permutations

Here objects are arrange in a circle

The number of ways of arranging n unlike objects in a ring when clockwise and anticlockwise are different.

Consider four people A, B, C and D seated at a round table. The possible arrangements are as shown below



With circular arrangements of this type, it is the relative positions of the objects being arranged which is important. The arrangements of the people above is the same. However, if the people were seated in a line the arrangements would not be the same, i.e. A, B, C, D is not the same as D, A, B, C. When finding the number of different arrangements, we fix one person say A and find the number of ways of arranging B, C and D.

Therefore, the number of different arrangements of four people around the table is 3!

Hence the number of different arrangements of n people seated around a table is (n – 1)!

Example 17

(a) Seven people are to be seated around a table, in how many ways can this be done
Solution
The number of ways = (7 – 1)! = 6!
= 720

(b) In how many ways can five people A, B, C, D and E be seated at a round table if
(i) A must be seated next to B

Solution

If A and B are seated together, they are taken as bound together. So four people are considered

The number of ways = (4 – 1)! = 3! = 6

The number of ways in which A and B can be arranged = 2

The total number of arrangements

= 6 x 2 = 12 ways

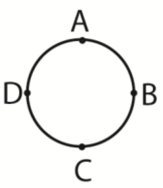
(ii) A must not seat next to B

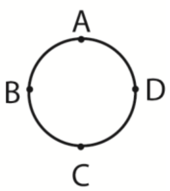
If A and B are not seated together, then the number of arrangements = total number of arrangements – number of arrangements when A and B are seated together

= (5 – 1)! – 12 = 12 ways

The number of ways of arranging n unlike objects in a ring when clockwise and anticlockwise arrangements are the same

Consider the four people above, if the arrangement is as shown below





Then the above arrangements are the same since one is the other viewed from the opposite side

The number of arrangements = $\frac{3!}{2}$ = 3ways

Hence the number of ways of arranging n unlike objects in a ring when clockwise and anticlockwise arrangements are the same = $\frac{(n-1)!}{2}$

Example 18

A white, a blue, a red and two yellow cards at arranged on a circle. Find the number of arrangements if red and white cards are next to each other.

Solution

If red and white cards are next to each other, they are considered as bound together. So we have four cards. Since anticlockwise and clockwise arrangements are the same and there are two yellow cards, the number of arrangements = $\frac{(4-1)!}{2\ x\ 2!} = \frac{3!}{2\ x\ 2!}$

The number of ways of arranging red and white cards = 2

Total number of ways of arrangements

=$\frac{3!}{2\ x\ 2!}\ x\ 2$ = 3

Revision exercise 1

1. In how many ways can the letters of the words below be arranged
   (a) Bbosa (5!]
   (b) Precious [8!]
2. How many different arrangements of the letters of the word PARALLELOGRAM can be made with A's separate [83160000]
3. How many different arrangements of the letters of the word CONTACT can be made with vowels separated? [900]
4. How many odd numbers greater than 6000 can be formed using digits 2, 3, 4, 5 and 6 if each digit is used only once in each number [12]

5. Three boys and five girls are to be seated on a bench such that the eldest girl and eldest boy sit next to each other. In how many ways can this be done [2 x 7!]
6. A round table conference is to be held between delegates of 12 countries. In how many ways can they be seated if two particular delegates wish to sit together [2 x 10!]
7. In how many ways can 4 boys and 4 girls be seated at a circular table such that no two boys are adjacent [144]
8. How many words beginning or ending with a consonant can be formed by using the letters of the word EQUATION? [4320]

Combinations

A combination is a selection of items from a group not basing on the order in which the items are selected

Consider the letters A, B, C, D

The possible arrangements of two letters chosen from the above letters are

AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB, DC. AS seen earlier, the total number of arrangements of the above letters is expressed as $\frac{4!}{(4-2)!} = \frac{4!}{2!} = 12$

However, when considering combinations, the grouping such as AB and BA are said to be the same groupings such as CA and AC, AD and DA, etc.

So the possible combinations are AB, AC, AD, BC, BD, CD which is six ways.

Thus the number of possible combinations of n items taken r at a time is expressed as ^nC_r or $\binom{n}{r}$ which is defined as $^nC_r = \frac{n!}{(n-r)!r!}$ where $r \leq n$

Hence the number of combinations of the above letters taken two at a time is

$$\binom{4}{2} = \frac{4!}{2!2!} = 6$$

Example 19

A committee of four people is chosen at random from a set of seven men and three women

How many different groups can be chosen if there is at least one

(i) Woman on the committee

Solution

Possible combinations

7 men	3 women
3 2 1	1 2 3

The number of ways of choosing at least one woman

$$=\binom{7}{3} x \binom{3}{1} + \binom{7}{2} x \binom{3}{2} + \binom{7}{1} x \binom{3}{3}$$

$$=\frac{7!}{3!4!} x \frac{3!}{1!2!} + \frac{7!}{2!5!} x \frac{3!}{2!1!} + \frac{7!}{1!6!} x \frac{3!}{3!0!} = 175$$

(ii) Man on the committee

7 men	3 women
1 2 3 4	3 2 1 0

The number of ways of choosing at least one man

$$\binom{7}{1} x \binom{3}{3} + \binom{7}{2} x \binom{3}{2} + \binom{7}{3} x \binom{3}{1} + \binom{7}{4} x \binom{3}{0}$$

$$=\frac{7!}{1!3!} x \frac{3!}{0!3!} + \frac{7!}{5!2!} x \frac{3!}{1!2!} + \frac{7!}{4!3!} x \frac{3!}{2!1!} + \frac{7!}{3!4!} x \frac{3!}{3!0!}$$

=210

Example 20

A group of nine has to be selected from ten men and eight women. It can consist of either five men and four women or four men and five women. How many different groups can be chosen?

Solution

Possible combination

10 men	8 women
5 4	4 5

Number of groups = $\binom{10}{5} x \binom{8}{4} + \binom{10}{4} x \binom{8}{5}$

= $\frac{10!}{5!5!}$ x $\frac{8!}{4!4!}$ + $\frac{10!}{6!4!}$ x $\frac{8!}{5!3!}$ = 29400

Example 21

A team of six is to be formed from 13 boys and 7 girls. In how many ways can the team be selected if it must consist of

(a) 4 boy and 2 girls

13 boys	7 girls
4	2

$\binom{13}{4}.\binom{7}{2}$= $\frac{13!}{9!4!} x \frac{7!}{5!2!}$ = 15015

(b) At least one member of each sex

Possible combinations

13 boys	7 girls
5	1
4	2
3	3
2	4
1	5

=$\binom{13}{5}.\binom{7}{1} + \binom{13}{4}.\binom{7}{2} + \binom{13}{3}.\binom{7}{3} + \binom{13}{2}.\binom{7}{4} + \binom{13}{1}.\binom{7}{5}$

=$\frac{13!}{8!5!}.\frac{7!}{6!1!} + \frac{13!}{9!4!}.\frac{7!}{5!2!} + \frac{13!}{10!3!}.\frac{7!}{4!3!} + \frac{13!}{11!2!}.\frac{7!}{3!4!} + \frac{13!}{12!1!}.\frac{7!}{2!5!}$

= 37037

Example 22

A team of 11 players is to be chosen from a group of 15 players. Two of the 11 are to be randomly elected a captain and vice-captain respectively. In how many ways can this be done?

Number of ways of choosing 11 players from 15 = $\binom{15}{11}$

A captain will be elected from 11 players and a vice-captain from 10 players

Total number of selection =$\binom{15}{11}$ x 11 x 10

=$\frac{15!}{4!11!}$ x 11 x 10 = 150150

Example 23

(a) Find the number of different selections of 4 letters that can be made from the word UNDERMATCH.
Solution
There are 10 letters which are all different
Number of selections of 4 letters from 10 is given by $\binom{10}{4} = \frac{10!}{(10-4)!4!} = \frac{10!}{6!4!} = 210$

(b) How many selections do not contain a vowel?
Solution
Number of vowels in the word = 2
Number of letters not vowels = 8
Number of selections of 4 letters from 10 without containing a vowel = selecting 4 letters from 8 consonants =
$\binom{8}{4} = \frac{8!}{(8-4)!4!} = \frac{8!}{4!4!} = 70$

Example 24

In how many ways can three letters be selected at random from the word BIOLOGY is selection

(a) Does not contain the letter O
Solution
Number of selections without the letter O = number of ways of choosing three letters from B, I, L, G, y
$= \binom{5}{3} = \frac{5!}{2!3!} = 10$

(b) Contain only the letter O

Solution

Number of selections with one letter O = number of ways of choosing two letters from B, I, L, G, y

$= \binom{5}{2} = \frac{5!}{3!2!} = 10$

(c) Contains both of the letters O

Solution

Number of selections with two letter O = number of ways of choosing one letter from B, I, L, G, y

$= \binom{5}{1} = \frac{5!}{4!1!} = 5$

Example 25

In how many ways can four letters be selected at random from the word BREAKDOWN if the letters contain at least one vowel?

Solution

Vowels: E, A, O (3)

Consonants: B, R, K, D, W, N (6)

Consonants (6)	Vowels (3)
3	1
2	2
1	3

Number of selection of four letters with at least one vowel

$$= \binom{6}{3}.\binom{3}{1} + \binom{6}{2}.\binom{3}{2} + \binom{6}{1}.\binom{3}{3} = 111$$

Example 26

How many different selections can be made from the six digits 1, 2, 3, 4, 5, 6

Solution

Note: this an open questions because selections can consist of only one digit, two digits, three digits, four digits, five digits or six digits

Number of selection of 1 digit = ${}^6C_1$ = 6

Number of selection of 2 digits = ${}^6C_2$ = 15

Number of selection of 3 digits = ${}^6C_3$ = 20

Number of selection of 4 digits = ${}^6C_4$ = 15

Number of selection of 5 digits = ${}^6C_5$ = 6

Number of selection of 6 digits = ${}^6C_1$ = 1

Total number of selections

= 6 + 15 + 20 + 15 +6 + 1 = 63

This approach is tedious for a large group of objects.

The general formula for selection from n unlike objects is given by $2^n - 1$.

For the above problems, number of selections = $2^6 - 1$ = 63

Example 26

How many different selections can be made from 26 different letters of the alphabet?

Number of selection = $2^{26} - 1$

= 67,108,863

Cases involving repetitions

Suppose we need to find the number of possible selections of letters from a word containing repeated letters, we take the selections mutually exclusive

Example 27

How many different selections can be made from the letters of the word CANADIAN?

Solution

There are 3A's, 2N's and 3 other letters

The A's can be dealt with in 4 ways (either no A, 1A's, 2A's or 3A's)

The N's can be dealt in 3 ways (no N, 1N, or 2N's)

The C can be dealt with in 2 ways (no C, 1C)

The D can be dealt with in 2 ways (no D, 1D)

The I can be dealt with in 2 ways (no I, 1I)

The number of selections

= 4 x 3 x 2 x 2 x 2 – 1 = 95

Example 28

How many different selections can be made from the letters of the word POSSESS?

Solution

There are 4S's and 3 other letters

The S's can be dealt in 5 ways (no S, 1S, 2N's, 3S's, 4S's, or 5S's)

The P can be dealt with in 2 ways (no P, 1P)

The O can be dealt with in 2 ways (no O, 1O)

The E can be dealt with in 2 ways (no E, 1E)

Total number of selections = 5 x 2 x 2 x 2 – 1

= 39

Cases involving division into groups

The number of ways of dividing n unlike objects into say two groups of p and q where p + q = n is given by $\frac{n!}{p!q!}$

For three groups of p, q and r provided p + q + r = n

Number of ways of division = $\frac{n!}{p!q!r!}$

However, for the two groups above, if p = q then the number of ways of division = $\frac{n!}{p!p!2!}$

For three groups where p = q = r

then the number of ways of division = $\frac{n!}{p!p!p!3!}$

Example 29

The following letters a, b, c, d, e, f, g, h, I, j, k, l are to be divided into groups containing

(a) 3, 4, 5
(b) 5, 7
(c) 6, 6
(d) 4, 4, 4 letters. In how many ways can this be done?

Solution

(a) Number of ways = $\frac{12!}{3!4!5!}$ = 27720
(b) Number of ways = $\frac{12!}{5!7!}$ = 792
(c) Number of ways = $\frac{12!}{6!6!2!}$ = 462
(d) Number of ways = $\frac{12!}{4!4!4!3!}$ =5775

Example 30

Find the number of ways that 18 objects can be arranged into groups if there are to be

(a) Two groups of 9 objects each
(b) Three groups of 6 objects each
(c) 6 groups of 3 objects each
(d) Three groups of 5, 6 and 7 objects each

Solution

(a) Number of ways = $\frac{18!}{9!9!2!}$ = 24310
(b) Number of ways = $\frac{18!}{6!6!6!3!}$ = 2858856
(c) Number of ways = $\frac{18!}{3!3!3!3!3!3!6!}$ =190590400
(d) Number of ways = $\frac{18!}{5!6!7!}$ = 14702688

Example 31

(a) Find how many words can be formed using all letters in the word MINIMUM.

Solution

Number of ways of arranging the letters = 7!

There are 3M's and 2I's

Number of words formed = $\frac{7!}{3!2!} = 420$

(b) Compute the sum of four-digit numbers formed with the four digits 2, 5, 3, 8 if each digit is used only once in each arrangement

Solution

Number of ways of arranging a four digit number = 4!

Sum of any four digit number formed = 2 + 5 + 3 + 8 = 18

Total sum of four digit numbers formed

= 18 x 4! = 432

(c) A committee consisting of 2 men and 3 women is to be formed from a group of 5 men and 7 women. Find the number of different committees that can be formed. If two of the women refuse to serve on the same committee, how many committees can be formed?

Solution

The committees formed = ${}^5C_2.{}^7C_3$

= 10 x 35 = 350

Suppose two women are to serve together, we take them as glued together, so the number of committees =${}^5C_2.{}^6C_3$ = 200

Number of committees in which two women refuse to serve together = 350 – 200 = 150

Revision exercise 2

1. (a) Find the number of different selection of 3 letters that can be made from the word PHOTOGRAPH. [53]
   (b) How many of these selections contain no vowel [18]
   (c) How many of these selections contain at least one vowel? [35]
2. (a) find the number of different selections of 3 letters that can be made from the letters of the word SUCCESSFUL.[36]
   (c) How many of these selections contain only consonants [11]
   (d) How many of these selections contain at least one vowel [25]
3. (a) Find the value of n if ${}^{n}P_4 = 30\,{}^{n}C_5$ [8]
   (b) How many arrangement can be made from the letters of the name MISSISSIPPI
      (i) when all the letters are taken at a time [34650]

      (ii) If the two letters PP begin every word [630 ways]

   (c) Find the number of ways in which a one can chose one or more of the four girls to join a discussion group [15 ways]
4. Find in how many ways 11 people can be divided into three groups containing 3, 4, 4 people each. [5775]
5. A group of 5 boys and 8 girls. In how many ways can a team of four be chosen, if the team contains
   (a) No girl [5]
   (b) No more than one girl [85]
   (c) At least two boys [365]
6. Calculate the number of 7 – letter arrangements which can be made with the letters of the word MAXIMUM. In how many of these do all the 4 consonants appear next to each other? [840, 96]
7. In how many ways can a club of 5 be selected from 7 boys and 3 girls if it must contain
   (a) 3 boys and 3 girls [105]
   (b) 2 men and 3 girls [21]
   (c) At least one girl [231]
8. How many different 6 digit numbers greater than 400,000 can be formed form the following digits 1, 4, 6, 6, 6 7? [100]

Thank you

Dr. Bbosa Science

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Mock Excel

Monday 16th October 2023 – Morning

Paper reference P425/ 1

Mathematics

Advanced

PAPER 1 : Pure

Time: 3hours

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

INSTRUCTIONS

- Answer **all** the **eight (8)** questions in section **A** and any **five (5)** from section **B**.
- Any additional question(s) answered will not be marked.
- All working must be shown clearly.
- Begin each answer on a fresh sheet of paper.
- Silent, non-programable scientific calculators and mathematical tables with a list of formulae may be used.

A	
B	

SECTION A (40 MARKS)

1. The council of Broxbourne undertook a housing development scheme which started in the year **2001** and is to finish in the year **2025**. Under this scheme the council will build **760** houses in **2012** and **240** houses in **2025**. The number of houses the council builds every year, forms an arithmetic sequence.
   a) Determine the number of houses built in **2001**.
   b) Calculate the total number of houses that will be built under this scheme. **(5)**
2. Solve the two pairs of simultaneous equations.
$$x^3 + 9x^2y = -28$$
$$y^3 + xy^2 = 1$$
(5)
3. The straight line l passes through the point **P(4,5)** and has gradient **3**. The point **Q** also lies on l so that the distance **PQ** is $3\sqrt{10}$. Determine the coordinates of the two possible positions of **Q**. **(5)**
4. Show that:
$$\int_0^2 \frac{4x^3 - 12x^2 - 22x - 3}{(4-x)(2x+1)}\,dx = \frac{1}{2}In\left(\frac{5}{64}\right) - 6$$
5. Use **De Moivre's theorem** to show that:
   $\sin 5\theta = \sin\theta(16\cos^4\theta - 12\cos^2\theta + 1)$, hence solve.
   $\sin 5\theta = 10\cos\theta\sin 2\theta - 11\sin\theta$, for $0 < \theta < \pi$. **(5)**
6. A committee of **3 people** is to be picked from **9 individuals**, of which **4 are women** and **5 are men**. One of the **4 women** is married to one of the **5 men**. The selection rules state that the committee must have at least a member from each gender and no married couple can serve together in a committee. Determine the **number of possible committees** which can be picked from these 9 individuals. **(5)**
7. A circle has centre at the origin and radius **R**. This circle fits wholly inside the circle with equation: $x^2 + y^2 - 10x - 24y = 231$. Determine the range of possible values of **R**. **(5)**
8. The surface area **A**, of a metallic cube of side length **x**, is increasing at the constant rate of $0.4521 cms^{-1}$. Find the rate at which the volume of the cube is increasing, when the cube's side length is **8 cm.** **(5)**

SECTION B (60)

9. Solve the system of simultaneous equations is given below

$x + y + z = 1$

$x^2 + y^2 + z^2 = 21$

$x^3 + y^3 + z^3 = 55.$ **(12)**

10. A scientist is investigating the population growth of farm mice. The number of farm mice N, t months since the start of the investigation, is modelled by the equation

$$N = \frac{600}{1 + 5e^{-0.25t}}$$

a) State the number of farm mice at the start of the investigation.

b) Calculate the number of months that it will take the population of farm mice to reach 455.

c) Show clearly that:

$$\frac{dN}{dt} = \frac{1}{4}N - \frac{1}{2400}N^2$$

d) Find the value of t when the rate of growth of the population of these farm mice is largest. **(12)**

11. $2x^2 + kx + 1 = 0.$

The roots of the above equation are a and b, where k is a non-zero real constant. Given further that the following two expressions;

$$\frac{\alpha}{\beta(1 + \alpha^2 + \beta^2)} \quad and \quad \frac{\beta}{\alpha(1 + \alpha^2 + \beta^2)}$$

are real, finite and distinct, determine the range of the possible values of k. **(12)**

12. Max is revising for an exam by practicing papers. He takes **3 hours and 20 minutes** to complete the first paper and **3 hours and 15 minutes** to complete the second paper.

It is assumed that the times Max takes to complete each successive paper are consecutive terms of a geometric progression.

a) **Assuming** this model, shows that Max will take approximately …

i. … **176 minutes** to complete the sixth paper.

ii. … **35 hours** to complete the first 12 papers.

Max aims to be able to complete a paper in under two hours.

b) Determine, by using logarithms, the minimum number of papers he needs to practice in order to achieve his **target** according to this model.

(12)

13. The points P and Q are two distinct points which lie on the curve with equation

$$\boldsymbol{y = \frac{1}{x}\ , x \in \mathbf{R}, x \neq 0}$$

P and Q are free to move on the curve so that the straight-line segment PQ is a normal to the curve at P. The tangents to the curve at P and Q meet at the point R.

Show that R is moving on the curve with Cartesian equation

$$\boldsymbol{(y^2 - x^2)^2 + 4xy = 0}$$ **(12)**

14. Relative to a fixed origin *O*, the straight-line *l* passes through the points ***A*(*a*, -3, 6)**, ***B*(2, *b*, 2)** and ***C*(3,3,0)** , where *a* and *b* are constants.

a) Find the value of *a* and the value of *b,* and hence find a vector equation of *l.* The points *P* and *Q* lie on the *l* so that $|\boldsymbol{OP}| = |\boldsymbol{OQ}|$ and ***POQ*= 90°.**

b) Find the coordinates of *P* and the coordinates of *Q*. **(12)**

15. Find, in exact surd form, the only real solution of the following trigonometric equation

$$\boldsymbol{\sin^{-1}(2x - 1) - \cos^{-1} x = \frac{\pi}{6}}$$

The rejection of any additional solutions must be fully justified. **(12)**

16. The curve C has equation

$$y = \frac{1}{x^3 - 9x^2 + 24x}$$

Sketch the graph of C. **(12)**

END

KIIRA COLLEGE BUTIKI

Uganda Advanced Certificate of Education

SUBSIDIARY MATHEMATICS

Paper 1
Lock down revision questions
SECTION A.

1. Solve the simultaneous equations

   $log_4(xy) = 1$

   $log_4\left(\frac{x}{y}\right) = 3$ ***(5 marks)***

2. The curve $y = x^2 - qx + p$ has a turning point at (3, 10), find the values of p and q. ***(5 marks)***

3. The table below shows the results obtained by six candidates in a typing competition based on speed and accuracy.

Candidate	***A***	***B***	***C***	***D***	***E***	***F***
Time (min.)	***55***	***35***	***45***	***40***	***60***	***45***
Number of errors	***18***	***12***	***10***	***9***	***5***	***3***

   By ranking the fastest and one with fewer errors as best, calculate the rank correlation coefficient and use it to comment on the relationship between typing speed and number of errors. ***(5 marks)***

4. Solve the differential equation $\frac{dy}{dx} = 2x - 7$, $given\ that\ \ x = 4\, when\, y = 3.$ ***(5 marks)***

5. Solve the equation $2cos^2x - 3sin2x = 0 \quad for \quad 0^0 \leq x \leq 360^0$ ***(5 marks)***

6. The weights of cabbages from a garden are said to be normally distributed with mean weight 2.4kg and standard deviation 0.5kg. Find the probability that a cabbage selected at random weighs between 1.8kg and 2.7 kg. ***(5 marks)***

7. Two events A and B are such that $p(A) = \frac{3}{4}$, $p(B) = \frac{4}{7}$ and $p(A \cap \bar{B}) = \frac{2}{5}$, find;

   (i) $p(A \cap B)$

   (ii) $p(A/B)$. ***(5 marks)***

8. A cyclist started from rest, accelerated uniformly for 3 minutes and then maintained a speed of $80kmh^{-1}$ for 15 minutes. He then decelerated uniformly for 2 minutes before coming to rest. How far has the cyclist travelled from the start? ***(5 marks)***

SECTION B.

9. **(a.)** The position vectors of points A, B and C are $2i + 3j$, $i - 5j$ *and* $10i + 7j$ respectively. Find angle ABC. ***(5 marks)***

(b.) Given matrices $A = \begin{pmatrix} 3 & -5 \\ 8 & 1 \end{pmatrix}$ *and* $B = \begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix}$.

find$AB - 3I$, *where I is a 2X2 identity matrix.* ***(4 marks)***

(c.) In a certain super market shs.3,3000 can buy 15 books and 6 pens and shs.1,0500 can buy 3 books and 9 pens. Use matrix method to find the cost of a book and a pen. ***(6 marks)***

10. The table shows the weights of animals on a certain exotic farm.

Weight (kg)	***Number of Animals***
80 ≤ X < 120	***8***
120 ≤ X < 160	***16***
160 ≤ X < 200	***39***
200 ≤ X <240	***30***
240 ≤ X < 280	***12***
280 ≤ X < 320	***5***

(a). Draw a histogram to illustrate the information above and estimate the modal weight.

(b) Calculate;

(i) Median.

(ii) Mean.

(iii) Standard deviation. ***(15marks)***

11. **(a).** The table below shows the consumption of some selected items in 2012 and 2015.

ITEMS	***2012***		***2015***	
	PRICE	***QTY***	***PRICE***	***QTY***
MEAT	***7100***	***10***	***8000***	***8***
MATOKE	***10000***	***15***	***12500***	***12***
BEANS	***1500***	***25***	***1600***	***20***
SUGAR	***1600***	***20***	***2500***	***22***
RICE	***2000***	***30***	***3500***	***40***

Using 2012 as your base year, calculate the weighted price index for the items in 2015 and comment on your result. ***(5marks)***

(b). The table below shows the Quarterly production of Sugar canes in thousands of tones in Busoga region for the period 2012 – 2014.

YEAR	***2012***	***2013***	***2014***
1ST QUARTER	***540***	***549***	***453***
2ND QUARTER	***516***	***402***	***318***
3RD QUARTER	***450***	***378***	***294***
4TH QUARTER	***519***	***411***	***351***

(i). construct a 4 point moving average.
(ii). Graph the moving averages together with the original data.
(iii). Comment on the trend of Sugarcane production over the 3 year period. ***(10marks)***

12. **(a).** In a game the probability that a player scores is $\frac{2}{3}$, five players were to play.

Find;
(i). Probability that at least 4 score.
(ii) Expected number of players who score. ***(7marks)***

(b). The discrete random variable x has probability density function

$$f(x) = \begin{cases} \frac{k}{x} ; & x = 1, 2 \\ kx ; & x = 3, 4, 5 \\ 0 ; & Otherwise \end{cases}$$

Find ; (i).Mode.
(ii). Mean of x. ***(8marks)***

13. **(a).** Given that $y = (x+2)^2 - 9$, find;
(i). intercepts.
(ii). Turning points.
(iii). Sketch the curve.

(b). find the area enclosed by the curve in (a)(iii). above and the x-axis. ***(15marks)***

14. **(a).** A force acting on a particle of mass 75kg moves it along a straight line with a velocity of $20ms^{-1}$. The rate at which work is done by the force is 40 watts. If the particle starts from rest, determine the time it takes to move a distance of 250m. ***(8marks)***

(b). A bus whose mass is 275mg moves up an inclined plane of 3 in 100 at a uniform rate of 48km/h. The resistance due to friction is equal to the weight of 4mg. At what power is the engine working? ***(7marks)***

S475/1
S.6 SUB – MATHEMATICS
Paper 1
July, 2019
$2\,^{2}/_{3}$ hours

Uganda Advanced Certificate of Education
INTERNAL MOCK EXAMINATIONS 2019
SUB – MATHEMATICS
Paper 1
2 hours 40 minutes

INSTRUCTIONS

- *Answer* **all** *the eight questions in section* **A** *and any* **four** *questions in section* **B.**
- *Any additional question(s) answered will* **not** *be marked.*
- *Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.*
- *Where necessary, take acceleration due to gravity,* $g = 9.8ms^{-2}$.

SECTION A (30 MARKS)

1. Express $\frac{4}{5+2\sqrt{2}} - \frac{5}{5-2\sqrt{2}}$ without a surd in its denominator. (05 marks)

2. A green, Red and three yellow balls are to be arranged in a row. What is the probability of having a red ball net to a green ball. (05 marks)

3. Solve the differential equation $4x\frac{dy}{dx} = 3$, given that $x = 1$, when $y = 2$. (05 marks)

4. The data below shows the temperature in degrees in a certain city at midday during the first week of May 12, 16, 14, 11, 12, 15, 13. Calculate the 3 point moving averages for the data. (05 marks)

5. Given that α and β are roots of the equation $x^2 + 5x + 6 = 0$. Form an equation whose roots are $\frac{1}{1-\alpha}$ and $\frac{1}{1-\beta}$ (05 marks)

6. A continuous random, variable X has a probability distribution below;

$$f(x) = \begin{cases} k(x^2 - 1): & 0 < x < 2 \\ 0 \quad ; & \text{otherwise} \end{cases}$$

Find;

(a) Value of constant k,

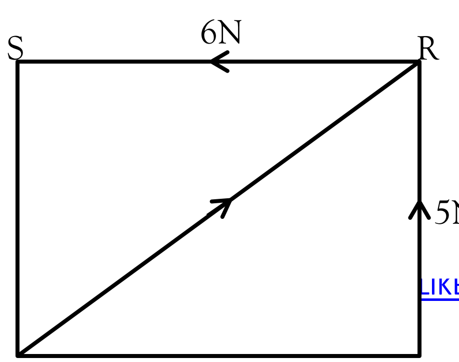
(b) $P(x > 1)$ (05 marks)

7. Given that $A = \begin{pmatrix} 2 & 1 \\ -1 & 0 \\ 3 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 2 \\ 0 & 1 \\ 3 & 4 \end{pmatrix}$. Find the value of

(a) $2B - A$

(b) order of matrix T if T$= 2B - A$. (05 marks)

8. PQRS is a square, forces of 3N, 5N, 6N, and $6\sqrt{2}$ N act along PQ, QR, RS, and PR respectively.



P $\overrightarrow{3N}$ Q

Calculate the resultant force of the system (05 marks)

SECTION B

9. The table below shows the prices and quantities of building materials between 2014 and 2015.

ITEM	PRICES IN UG SHS		QUANTITIES	
	2014	2015	2014	2015
Iron sheets	45,000	55,000	8	5
Cemment	28,000	35,000	12	18
Sand	140,000	105,000	10	10
Tiles	30,000	37,000	4	3
Nails	3,500	4,800	22	15

Taking 2014 as the base year, Calculate

a) Price relative for each item. (05 marks)
b) Simple average index for 2015 (04 marks)
c) Weighted aggregate price index for 2015 and hence comment on your result. (06 marks)

10. (a) Solve for θ : $\sin\theta - 2 = 2cos^2\theta - 3$, for $0 \leq \theta \leq 360^0$ (07 marks)

(b) If $\sin A = \frac{24}{25}$ and A is obtuse, and $\tan B = \frac{3}{4}$ and B is acute. Find without using tables or calculator,

(i) $Sin\ (A + B)$ (04 marks)
(ii) $\tan(A - B)$ (04 marks)

11. (a) Events A and B are independent such $P(A) = ⅓$ and $(B) = ¾$. Find the probability that

(i) both A and B occur (03 marks)
(ii) only one of the events occur (04 marks)

(b) A random variable X has a probability distribution function given below.

x	-1	0	1	2
$P(X = x)$	0.1	0.3	0.4	0.2

(a) Calculate;

(i) the mean mark (03 marks)
(ii) the standard deviation (04 marks)

12. (a) Find and determine the nature of the turning point of the curve $y = 5 + 8x - 4x^2$.
Hence sketch the curve. (10 marks)
(b) Calculate the area bounded by the curve $y = 5 + 8x - 4x^2$ and line $y = 5$. (05 marks)

13. The table below shows the marks scored by 50 students in a class.

Marks	$-< 30$	$-< 40$	$-< 50$	$-< 60$	$-< 70$	$-< 80$
Number of students	3	11	29	40	47	50

(a) Calculate;
(i) the mean mark (03 marks)
(ii) the standard deviation (04 marks)

(b) Display the data on an ogive and use it to estimate the:-
(i) median mark (02 marks)
(ii) range of middle 50 percentile mark. (03 marks)

14. Body of mass 8kg rests on a rough horizontal table and is connected to another body of mass 3kg by a light inextensible string passing over a smooth pulley fixed at the edge of the table. The 3kg mass hangs freely. The coefficient of friction between the 8kg mass and the table is 0.3. The system is released from rest. Find the
(i) acceleration of the system. (05 marks)
(ii) tension in the string (05 marks)
(iii) distance the 3 kg mass covers after 2 seconds. (05 marks)

END

PURE MATHEMATICS

Qn 1: Simplify the following using the laws of indices:

(a). $16^{\left(\frac{3}{4}\right)n} \div 8^{\left(\frac{5}{3}\right)n} \times 4^{(n+1)}$.

(b). $\frac{x^{(2n+1)} \times x^{\frac{1}{2}}}{\sqrt{x^{3n}}}$.

(c). $\frac{\sqrt{(xy)} \times x^{\frac{1}{3}} \times 2y^{\frac{1}{4}}}{(x^{10}y^{9})^{\frac{1}{12}}}$.

Qn 2: Using the laws of indices, evaluate:

(a). $(0.04)^{\frac{-3}{2}}$

(b). $\frac{4^{\frac{-3}{2}}}{8^{\frac{-2}{3}}}$.

Qn 3: Show that $\frac{3\sqrt{2}-4}{3-2\sqrt{2}} = \sqrt{2}$.

Qn 4: Solve the equation:

$$2^{x} \times 2^{x+1} = 10$$

Qn 5: Simplify:

(a). $-\log_2\left(\frac{1}{2}\right)$.

(b). $\frac{\log 49}{\log 343}$.

Qn 6: Given that $\log_{10} 2 = 0.3010$,

(a). Show that $\log_{10} 5 = 0.6990$.

(b). Hence find the value of $\log_2 5$.

Qn 7: Show that $\log_a b \times \log_b a = 1$. Hence evaluate: $\log_3 8 \times \log_2 9$.

Qn 8: The roots of the equation $x^2 + 6x + q = 0$ are α and $\alpha - 1$.
Find the value of q.

Qn 9: The roots of the equation $2x^2 + 3x - 4 = 0$ are α and β.
Find the values of
(a). $(\alpha + 1)(\beta + 1)$.

(b). $\frac{\beta}{\alpha} + \frac{\alpha}{\beta}$.

(c). $\alpha^2 + \alpha\beta + \beta^2$.

Qn 10: The roots of the equation $2x^2 - 4x + 1 = 0$ are α and β.
Find the integral coefficients whose roots are:
(a). α^2 and β^2.

(b). $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Qn 11: The roots of the equation $x^2 + px + q = 0$ differ by 2.
Show that $p^2 = 1 + q$.

Qn 12: Find the values of a and b if $ax^4 + bx^3 - 8x^2 + 6$ has remainder $(2x + 1)$ when divided by $x^2 - 1$.

Qn 13: $(x - 1)$ and $(x + 1)$ are factors of the expression $x^3 + ax^2 + bx + c$, and it leaves a remainder of 12 when divided by $(x - 2)$. Find the values of a, b and c.

Qn 14: Given the quadratic equation $ax^2 + bx + c = 0$,
Show that $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
Hence use it to solve the equation $(2 + x)(3 - 4x) = 0$.

Qn 15: By completing squares, find the greatest value of $-7 + 12x - 3x^2$ and the value of x at which it occurs.

Qn 16: (a). Find the turning points of the curve $y = (2 + x)(1 + x)(3 - x)$.

(b). Hence distinguish the above turning points.

Qn 17: The tangent to the curve $y = ax^2 + bx + c$ at the point where $x = 2$ is parallel to the line $y = 4x$. Given that y has a maximum value of -3 where $x = 1$, find the values of a, b and c.

Qn 18: (a). Sketch the graph of the curve $y = 7 - x - x^2$.
(b). Hence calculate the area bounded between the curve and the line $y = 5$.

Question 19:
The velocity v of a point moving along a straight line is given in terms of the time, t by the formula $v = 2t^2 - 9t + 10$, the point being at the origin when $t = 0$.
(a). Find the expression in terms of t for the distance from the origin.
(b). Find the expression in terms of t for the acceleration.
(c). Show that the point is at rest twice and find its distances from the origin at those instants.

Question 20:
(a). $\int \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right) dx$.
(b). Find s as a function of t given that $\frac{ds}{dt} = 6t^2 + 12t + 1$ and when $t = -2$, $s = 5$.

Question 21:
(a). Find the first term of a geometric series that has a common ratio of $\frac{3}{5}$ and a sum to infinity of 8.
(b). The first term of an A.P is -12 and the last term is 40. If the sum of the progression is 196, find:
(i). the number of terms,
(ii). the common difference.

Question 22:

(a). The first term of a G.P is 16 and the fifth term is 9. What is the value of the seventh term?

(b). The sum of the first terms of a G.P is 9 and the sum to infinity of the G.P is 25. If the G.P has a positive common ration r, find:

(i). r,

(ii). the first term.

Question 23:

(a). Find the possible values x can take given that

$$\boldsymbol{A} = \begin{pmatrix} x^2 & 3 \\ 1 & 3x \end{pmatrix},\ \boldsymbol{B} = \begin{pmatrix} 3 & 6 \\ 2 & x \end{pmatrix} \text{ and } \boldsymbol{AB} = \boldsymbol{BA}.$$

(b). If $\boldsymbol{A} = \begin{pmatrix} 3 & 2 \\ -4 & 1 \end{pmatrix}$, find the values of m and n given that $\boldsymbol{A}^2 = m\boldsymbol{A} + n\boldsymbol{I}$ where $\boldsymbol{I}$ is a 2×2 identity matrix.

(c). Solve the following equations using the matrix method.

$$x - 3y = 3$$
$$5x - 9y - 11 = 0$$

Question 24:

(a). Find the inverse of the matrix $\boldsymbol{M}$ where $\boldsymbol{M} = \begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix}$ and hence solve the matrix equation $\boldsymbol{MX} = \boldsymbol{C}$ in which $\boldsymbol{X} = \begin{pmatrix} x \\ y \end{pmatrix}$ and $\boldsymbol{C} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

(b). Find the values of x for which the matrix $\begin{pmatrix} x-2 & 1 \\ 2 & x-3 \end{pmatrix}$ has no inverse.

(c). Calculate the inverse of $\boldsymbol{M}^{-1}$ of the matrix $\boldsymbol{M} = \begin{pmatrix} x & x-1 \\ y & y \end{pmatrix}$, where $y \neq 0$.

Find the values of x and y such that $\boldsymbol{M}^{-1}\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$.

Question 26:

Solve the following equations for all values of θ from $0°$ to $360°$.

(i). $\sin(\theta + 20)° = \frac{-\sqrt{3}}{2}$

(ii). $4 - \sin\theta = 4\cos^2\theta$.

(iii). $3\tan^2\theta + 5 = 7\sec\theta$.

Question 27:
If $\cos\theta = \frac{-8}{7}$ and θ is obtuse, find without using tables or calculators, the value of
(i). $\sin\theta$,
(ii). $\cot\theta$.

Question 28:
If $\tan\theta = \frac{7}{24}$ and θ is reflex, find without using tables or calculators, the value of
(i). $\sec\theta$,
(ii). $\sin\theta$.

Question 29:
If $\sin\theta = \frac{3}{5}$, find without using tables or calculator, the values of
(i). $\cos\theta$,
(ii). $\tan\theta$.

Question 30:
Prove the following identities:
(a). $\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta}$.

(b). $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$.

(c). $\frac{1-\cos^2\theta}{\sec^2\theta - 1} = 1 - \sin^2\theta$.

Question 31:
If $\sin A = \frac{3}{5}$ and $\sin B = \frac{5}{13}$, where A and B are acute angles, find the values of
(a). $\sin(A + B)$
(b). $\cos(A + B)$
(c). $\tan(A + B)$

Question 32:
If $\sin(x - \alpha) = \cos(x + \alpha)$, prove that $\tan x = 1$.

Question 33:

If $\sin A = \frac{4}{5}$ and $\cos B = \frac{12}{13}$, where A is obtuse and B is acute, find the values of

(a). $\sin(A - B)$

(b). $\tan(A - B)$

(c). $\tan(A + B)$.

Question 34:

(a). The points E, F and G have position vectors $\left(2\underset{\sim}{\boldsymbol{i}} + 2\underset{\sim}{\boldsymbol{j}}\right)$, $\left(\underset{\sim}{\boldsymbol{i}} + 6\underset{\sim}{\boldsymbol{j}}\right)$ and $\left(-7\underset{\sim}{\boldsymbol{i}} + 4\underset{\sim}{\boldsymbol{j}}\right)$. Show that the triangle EFG is right angled at F.

(b). If the angle between the vectors $\underset{\sim}{\boldsymbol{c}} = a\underset{\sim}{\boldsymbol{i}} + 2\underset{\sim}{\boldsymbol{j}}$ and $\underset{\sim}{\boldsymbol{d}} = 3\underset{\sim}{\boldsymbol{i}} + \underset{\sim}{\boldsymbol{j}}$ is $45°$, find the possible values of a.

Question 35:

(a). The points A, B, C and D have position vectors $\left(5\underset{\sim}{\boldsymbol{i}} + \underset{\sim}{\boldsymbol{j}}\right)$, $\left(-3\underset{\sim}{\boldsymbol{i}} + 2\underset{\sim}{\boldsymbol{j}}\right)$, $\left(-3\underset{\sim}{\boldsymbol{i}} - 3\underset{\sim}{\boldsymbol{j}}\right)$ and $\left(\underset{\sim}{\boldsymbol{i}} - 6\underset{\sim}{\boldsymbol{j}}\right)$. Show that AC is perpendicular to BD.

(b). Find a vector that is of magnitude 2 units and is parallel to $\left(4\underset{\sim}{\boldsymbol{i}} - 3\underset{\sim}{\boldsymbol{j}}\right)$.

(c). The point $\boldsymbol{A}$ has position vector $\left(3\underset{\sim}{\boldsymbol{i}} + \underset{\sim}{\boldsymbol{j}}\right)$ and point $\boldsymbol{B}$ has position vector $\left(10\underset{\sim}{\boldsymbol{i}} + \underset{\sim}{\boldsymbol{j}}\right)$. Find the position vector of the point which divides $\boldsymbol{AB}$ in the ratio $3:4$.

Question 36:

The points A, B and C have position vectors $\underset{\sim}{\boldsymbol{a}}$, $\underset{\sim}{\boldsymbol{b}}$ and $\underset{\sim}{\boldsymbol{c}}$ respectively referred to an origin O.

(a). Given that the point X lies on AB produced so that $AB: BX = 2:1$, find x, the position vector of X in terms of $\underset{\sim}{\boldsymbol{a}}$ and $\underset{\sim}{\boldsymbol{b}}$.

(b). If Y lies on BC between B and C so that $BY: YC = 1:3$, find y, the position vector of Y in terms of $\underset{\sim}{\boldsymbol{b}}$ and $\underset{\sim}{\boldsymbol{c}}$.

(c). Given that Z is the midpoint of AC, show that X, Y and Z are collinear.

(d). Calculate $XY: YZ$.

STATISTICS AND PROBABILITY

Question 1:
The table below represents the weights of 50 patients who visited a certain health centre in May 2019.

Weights (kg)	Percentage frequency (%)
40 – 50	10
50 – 60	16
60 – 70	24
70 – 80	30
80 – 90	14
90 – 100	6

(a). Calculate the:
  (i). mean weight,
  (ii). modal weight,
  (iii). number of patients who weigh between 55 kg and 85 kg.
(b). Draw a cumulative frequency curve and use it to estimate the:
  (i). median weight,
  (ii). interquartile range of weight.

Question 2:
The table below shows the number of pupils who failed to hand in the Maths homework each day and the minutes of yoga the teacher does to calm himself down.

Pupils missing homework (X)	3	5	2	10	2	0	4	8	15	6	1	4
Minutes of yoga (Y)	10	12	9	25	8	3	15	20	26	10	7	10

(a). Draw a scatter diagram and line of best fit to show the information.
(b). (i). If 7 pupils forget to hand in their homework, how many minutes of yoga might the teacher do?
  (ii). If the teacher did 28 minutes of yoga, how many pupils might have forgotten their homework?

(c). Calculate the rank correlation coefficient and comment on the level of significance.

Question 3:
A construction company uses five items of materials for a particular construction job. The table below represents the prices in shillings for each material used in the job for the year 2017 and 2019.

Materials	Price in 2017	Price in 2019	Weight
Sand	10,500	18,424	3
Cement	16,750	18,424	2
Water	1,250	1,500	4
Lime	8,200	10,250	1
Paste	3,750	5,250	1

Given that 2017 is assigned as an index of 100,
(i). Construct the unweighted price relative index for each material.
(ii). construct the simple average price relative for the material.
(iii). construct the weighted aggregate index for the materials.

Question 4:
The table below shows the sales in units of each of the four quarters for the years 2003, 2004 and 2005.

Years	Quarters			
	1^{st}	2^{nd}	3^{rd}	4^{th}
2003	600	840	420	720
2004	640	860	420	740
2005	670	900	430	760

(a). Calculate the four point moving averages.
(b). On the same axes, plot graphs of sales and the moving averages against time. Comment on the general trend of the sales for the three years' period.
(c). Use your graph to estimate the sales in:
  (i). the 4^{th} quarter of 2002.
  (ii). the 1^{st} quarter of 2006.

Question 5:

(a). A die and a coin are tossed once, what is the probability of getting a five on a die and a head on a coin?

(b). A student who wishes to join a higher institution of learning after A-level has the following choices with their corresponding probabilities.

Institution	**Probability**
M.U.B.S	04
Mbale	0.2
Nkozi	0.1
Nkumba	0.2
U.C.U	0.1

(i). Find the probability that he joins either M.U.B.S or Nkumba.
(ii). State, with a reason, the type of events above.

(c). In a certain school, there are 50 Senior five students of which 10 are girls; and 30 Senior six students of which 5 are girls. A student is to be picked at random to represent the school in a mathematics contest.
(i). Find the probability that the student chosen is a girl.
(ii). If the selected student is a girl, what is the probability that she is in Senior six?

Question 6:
In how many ways can the letters of the word MISSISSIPI be arranged:
(i). without restriction.
(ii). if the I's are kept together.
(iii). if the I's are separated.
(iv). if all the I's are the end of every word.
(v). if every word starts with M and ends with P.

Question 7:
An organizing committee of 4 people is to be chosen from 3 women and 4 men to organize a school end of year party. Determine the probability that the school with consist of at most 2 women.

Question 8:
A discrete random variable X has a probability density function $f(x)$, defined by

$$f(x) = \begin{cases} kx^3 & ; \quad x = 0, 1, 2, 3, \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

Where k is a constant.
Determine :
(i). the value of k,
(ii). $P((1 \leq X < 3)/X > 0)$,
(iii). $E(X)$,
(iv). $Var\ (X)$

Question 9:
In a certain clinic, 20% of the patients who are treated with a certain drug often develop complications. If a random sample of 50 patients are treated with this particular drug, find the mean, variance and standard deviation of patients who will develop the complication.

Question 10:
A continuous random variable X has a probability density function $f(x)$, defined by

$$f(x) = \begin{cases} k(x^2 - 1) & ; \quad 0 \leq x \leq 2, \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

Where k is a constant.
Determine the:
(i). value of k,
(ii). $P(X > 1)$,
(iii). mean value of X,
(iv). variance of X.

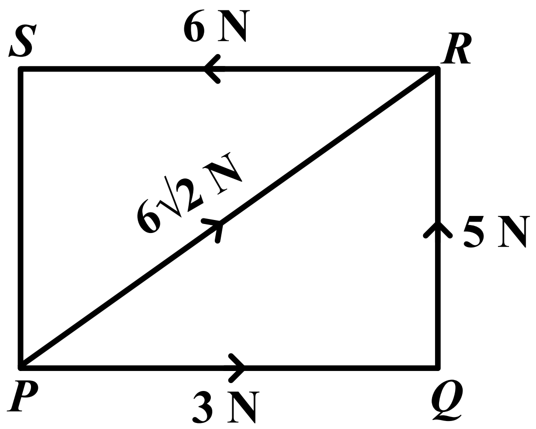
Question 11:
X is a random variable of a random experiment following a normal distribution with mean 40 and standard deviation 8.5. Determine:
(i). $P(X < 42.5)$,
(ii). $P(X > 31.5)$,
(iii). $P(31.5 < X < 36.8)$.

MECHANICS

Question 1:
A car initially moving at a speed of 80 m s^{-1} decelerates uniformly and attains a velocity of 40 m s^{-1} for 20s and comes to rest in the next 30 s. Sketch a velocity – time graph and use it to calculate the average velocity.

Question 2:
Forces of magnitude $2\sqrt{2}$ N, 4 N and 6 N, act on a body at angles 40°, 240° and 330° with the positive x −axis. Draw a clear force diagram and use it to find the magnitude of the resultant force and its direction to the x −axis.

Question 3:
PQRS is a square, forces of 3N, 5N, 6N, and $6\sqrt{2}$ N act along PQ, QR, RS, and PR respectively.

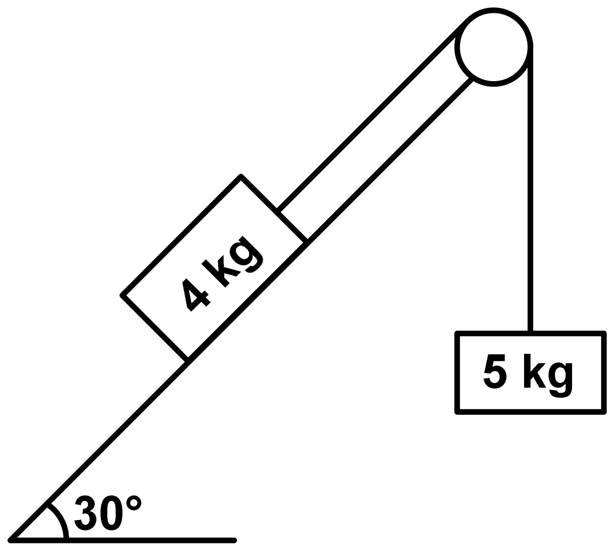


Calculate the resultant force of the system.

Question 4:
Two particles A and B of masses 3 kg and 5 kg are connected by a fine string passing over a smooth fixed pulley. Find:
(i). their common acceleration.
(ii). the tension in the string.

Question 5:
Body of mass 8 kg rests on a rough horizontal table and is connected to another body of mass 3 kg by a light inextensible string passing over a smooth pulley fixed at the edge of the table. The 3 kg mass hangs freely. The coefficient of friction between the 8 kg mass and the table is 0.3. The system is released from rest. Find the
(i). acceleration of the system.
(ii). tension in the string.
(iii). distance the 3 kg mass covers after 2 seconds.

Question 6:
The diagram below shows two masses of 4 kg and 5 kg attached to each other by a light inextensible string which passes over a smooth pulley fixed at the top of the wedge.



(a). Find the coefficient of friction between the 4 kg mass and the wedge if the system is in limiting equilibrium.
(b). Calculate the acceleration of the system if the 5 kg mass is increased by 1 kg.

Question 7:
A particle of mass 3 kg slides from rest down a rough plane inclined at 30° to the horizontal. Given that the coefficient of friction between the particle and the plane is 0.3, find the:
(i). normal reaction of the plane
(ii). frictional force exerted on the body
(iii). acceleration of the particle
(iv). how far it travels in 4 seconds.

Question 8:
A force of magnitude 40 N acts on a body causing it to change its velocity from 4 m s^{-1} to 10 m s^{-1} after 5 seconds. Find the work done by the force.

Question 9:
A lorry of mass 200 kg travels against friction resistance of 2600 N for 50 m along a level road at a constant speed of 45 km h^{-1}. Calculate the:
(i). work done by the engine,
(ii). power at which the engine is working.

Question 10:
A heavy truck of mass 10 tonnes has an engine which exerts a maximum power of 20 kW. The truck moves on a level ground against a constant resistance force F newton.
(a). If the truck attains a maximum speed of 57.6 km h^{-1}, find the value of F.
(b). Given that the truck ascends an incline of 1 in 100, find its maximum speed. (Assume resistance remains the same as in (a) above)

Question 11:
A rough plane is inclined at an angle $\sin^{-1}\left(\frac{3}{5}\right)$ to the horizontal.
A body of mass 20 kg is released from rest at the top of the plane to slide down the whole length of the plane in 5 s. If the coefficient of friction between the plane and the body is 0.5, calculate the:
(a). length and height of the inclined plane,
(b). loss in potential energy of the mass during the slide down period,
(c). velocity at the time the mass is at the bottom end of the inclined plane.

END

UGANDA ADVANCED CERTIFICATE OF EDUCATION

JAM MOCK EXAMINATIONS

SUBSDIARY MATHEMATICS

Paper 1

2hours 40minutes

INSTRUCTIONS

answer **all** the eight questions in **section A** and any **four** from **section B**

all the necessary working must be clearly shown

each question in **section A** is of **5 marks** while each question in **section B** carries **15 marks**

begin each number on a fresh sheet of paper

graph papers are provided

silent non-programmable calculators may be used

mathematical tables and a list of formulae are provided

where necessary, use $g = 9.8ms^{-2}$

SECTION A (40 MARKs)

Answer all the questions from this section

1. Given that $\dfrac{3^4 \times 3^8}{9 \times 3^7} = 3^{2x}$, find the value of x
(05marks)

2. Find $\mathbf{AB} + 3\mathbf{I}$ when $\mathbf{A} = \begin{pmatrix} -3 & 1 \\ 2 & 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 7 & 0 \\ 8 & -6 \end{pmatrix}$ and $\mathbf{I}$ is a 2×2 identity matrix
(05marks)

3. Find the value of y at the minimum point of the curve $y = x^2 - 6x + 8$
(05marks)

4. If $\cos x = a$ find in terms of a,
(i) $\sec x$

(ii) $\sin x$ (05marks)

5. The time in minutess for phone calls made by twelve customers at public telephone booth were recorded as follows:
20, 42, 11, 21, 9, 12, 15, 16, 19, 12, 15, 16

Find the: (**a**) median time (**b**) mean time
(05marks)

6. A random variable X has probability distribution function given in the table below:

x	0	2	3	4
$P(X = x)$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{1}{5}$	$\frac{2}{5}$

Find the (i) expectation $E(X)$ (ii) variance of X
(05marks)

7. The price in shillings of commodities A, B and C in 2012 and 2013 are given in the table below:

Commodity	2012	2013
A	,500	750
B	1500	2100
C	10,000	12000

Using 2012 as the base year, find the
a. Price relative for each commodity
b. Average price index
(05marks)

8. A car starts moving at a speed of $u\ ms^{-1}$ and moves at constant acceleration of $3ms^{-2}$ for 5seconds until it attains a speed of $v = 126\ kmh^{-1}$. Find the:

a. value of v in ms^{-1}
b. value of u
(05marks)

SECTION B (60 Marks)

*Answer only **four** questions from this section*

9. A continuous random variable X has the pdf $f(x)$ where

$$f(x) = \begin{cases} k(2x+1); & 1 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

a. Find the
(i) value of k
(04marks)
(ii) expectation of X
(04marks)
(iii) variance of X
(05marks)

b. Sketch the graph of the of $f(x)$
(02marks)

10. **a**. Find the number of terms in the Arithmetic progression (A.P)
$-10 - 2 + 6 + \ldots + 70$ (07marks)

b. An Arithmetic Progression (A.P) has a common difference -3. The sum of the first twenty terms is ten times the second term. Find the sum of the first 15 terms of the A.P.
(08marks)

11. The marks obtained by a student in 10 tests of History (x) and Literature (y) were as given in the table below:

x	45	47	65	80	70	55	60	44	35	80
y	75	84	60	37	50	65	75	96	90	50

(a) Calculate the rank correlation coefficient (05marks)
(b) Comment on your answer in (a) above (01marks)
(c) Draw a scatter diagram for the data (04marks)
(d) Draw a line of best fit (03marks)
(e) Use the line of best fit to find mark that the student would get in History if he got 54 in Literature (02marks)

12. **a.** Solve the equation $3x^2 - x - 24 = 0$ (04marks)

b. Form a quadratic equation whose roots are $\frac{3}{5}$ and -3
(04marks)

c. The roots of the quadratic equation $2x^2 - 3x - 6 = 0$ are α and β. Form a quadratic equation with integral coefficients whose roots are $\alpha - 2$ and $\beta - 2$
(07marks)

13. **a.** Two events A and B are such that
$P(A) = 0.4$, $P(B) = 0.7$, $P(A \cap B) = 0.5$
Find the $P(A \cup B)'$
(04marks)

b. Two independent events M and N are such that P(M) = 0.3 and P(N) = 0.4

Find the $P(M \cup N)$

(04marks)

c. A box contains 5 red and 8 blue pens. Two pens are picked at random one after the other without replacement. Find the probability that:

(i) both pens are red

(ii) the pens are of different colours

(07marks)

14. A motorist accelerates uniformly at a rate of $a\ ms^{-2}$ for 10s and then travels at a constant speed for 20s and slows down to rest at a constant retardation of $2a\ ms^{-2}$. If the total distance is 1100m

(i) sketch the velocity – time graph of the motion of the motorist

(ii) find the value of a

(iii) find the maximum speed attained by the motorist

(iv) find the retardation of the motorist

(15marks)

END

SECTION A (40 Marks)

Answer all questions in this section.

1. If $3log_t(10+3t)=6$, find the value of t. ***(5 marks)***

2. Given the polynomial $P(x) = 2x^3 + ax^2 + bx - 36$ where $p(3) = 0$ and $P(2) = 26$. Find the values of a and b. ***(5 marks)***

3. The table below shows the number of text books owned by 11 students of a certain class and their total marks in an exam.

	A	B	C	D	E	F	G	H	I	J	K
Number of text books	5	8	2	9	7	5	3	10	1	4	6
Total marks	290	370	184	366	366	277	190	385	200	281	331

Calculate the coefficient of rank correlation between the number of text books and the total marks.

Comment on your result. ***(5 marks)***

4. Solve the equation $sec^2\theta - 2sec\theta + 15 = 0$ ***(5 marks)***

5. Three bags A, B and C, each contain fruits Mangoes, Oranges and Apples as follows;

	Bag A	Bag B	Bag C
Mangoes	3	2	5
Oranges	1	3	6
Apples	3	3	4

A bag is chosen at random and then a fruit is picked from the selected bag at random

Determine the probability that the fruit picked is

(i.) An orange

(ii.) Not an orange. ***(5 marks)***

6. A discrete random variable X is represented by the probability function

$$P(X=x) = \begin{cases} \dfrac{1+i}{ik} & ; i = 1, 2, 3, \ldots .6 \\ 0 & ; \quad elsewhere \end{cases}$$

Find the ; (i.) value of k

(ii.) expectation of x ***(5 marks)***

7. A game coach has to choose a team of six out of ten of the best athletes, and one of the six has to be made a captain. How many possible teams can the coach obtain? *(5 marks)*

8. A hummer of mass 4.5 kg falls through a vertical height of 1 m and hits a nail of mass 50 grams directly without rebounding. If the nail is then driven into a piece of wood to a depth of 2 cm, find the common velocity of the hammer and nail just after impact. *(5 marks).*

SECTION B (60 MARKS)

Answer any four questions

9. (a.) The vectors $\boldsymbol{m} = \binom{5}{3}$ and $\boldsymbol{n} - \binom{-6}{p}$ are perpendicular to each other. Determine;

   (i.) The value of p

   (ii.) $|2m + n|$ *(8 marks)*

   (b.) Use matrix method to solve the simultaneous equations

   $3x^2 + 5y = 2$

   $2x^2 - 3y = 14$ *(7 marks)*

10. The gradient of curve at point P(x, y) is 4x + 3. If point A(3, 25) lies on the curve

    (i.) Find the equation of the curve

    (ii.) Determine the coordinates and nature of its turning point

    (iii.) Find the area enclose by the curve and the x- axis. *(15 marks)*

11. The sales of soda (in crates) at a certain canteen, open five days a week, were as follows:-

week	Mon.	Tue.	Wed	Thur.	Fri.
1	142	177	213	171	138
2	125	172	191	170	131
3	114	158	192	155	127

(a.) Plot the data, together with the five- point moving averages.

(b.) Comment on the trend of the sales of soda

(c.) Estimate the sales (in crates) on Monday of the 4th week. *(15 marks)*

12. (a.) if x is a normal random variable with mean 4 and variance 1.44. Find;

    (i.) $p(x > 5.3)$

    (ii.) $P(2.8 < x < 4.6)$ *(7 marks)*

(b.) The life time of a certain type of component of a computer is normally distributed with an average life of 1120 hours of working and standard deviation of 20 hours. Determine the probability that a component lasts

(i.) for less than 1150 hours

(ii.) Between 1115 hours to 1145 hours *(8 marks)*

13. The table below shows the age (at last birthday) at which expectant mothers in Jinja Hospital married

Age(in years)	16 – 20	20 – 24	25 – 29	30 – 34	35 – 39	40 – 44	45 – 49
Number of expectant mothers	60	120	80	30	3	20	10

(a.) Calculate

(i.) The mean age,

(ii.) The modal age , at which the expectant mothers married. *(7 marks)*

(b.) Draw an Ogive and use it to estimate;

(i.) The number of mothers who were 42 or over when they married.

(ii.) The number of mothers who were 18 or under when they were married.

(iii.) The range of the age of mothers for the middle 40% age bracket.

(8 marks)

14. A particle starts from rest, moving with a constant acceleration of 1.5 ms^{-1} for 30s.For the next 60s, the acceleration is 0.3 ms^{-1} and for the last $25s^{-1}$, it decelerates uniformly to rest.

(a.) Sketch the velocity time graph for the motion of the particle *(5 marks)*

(b.) Find the acceleration of the particle during the last period of motion *(3 marks)*

(c.) Determine the total distance travelled by the particle *(4 marks)*

(d.) Find the average speed for the whole journey. *(3 marks)*

Attempt **all** *questions in this section.*

1. Three matrices P, Q and I are such that P = $\begin{pmatrix} a & a+1 \\ a-1 & a+2 \end{pmatrix}$ is singular and I is an identity matrix. Find the value of $\boldsymbol{a}$ and hence the matrix **Q** if P + I = Q.

(05marks)

2. Given that A(1,2) B(4,3) and C(5, $^-$1) are vertices of a triangle ABC, find angle ABC.

(05marks)

3. If $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $4x^2 - 8x + 1 = 0$, find the equation whose roots are α and β. `

(05marks)

4. Two bags contain similar balls. Bag **A** contains 4 red and 3 white balls while bag **B** contains 3 red and 4 white balls. A bag is selected at random and a ball is drawn from it. Find the probability that a red ball is drawn.

(05marks)

5. When a polynomial $g(x)$ is divided by $x^2 + 2x - 3$, the remainder is $2x - 2$. Find the remainder when $g(x)$ is divided by;

$x - 1$

$x + 3$ (05marks)

6. The table below shows the price per kg of three food crops.

Item	Price per kg (shs)		Weights
	2000	**2010**	
Beans	4000	5000	3
Millet	3000	4000	3
Maize	2500	3000	4

i) Calculate the price index of each item for 2010 basing on 2000. (03marks)

ii) Calculate the weighted price index for 2010. (02marks)

7. The number of computers sold by **JA** Company in a period of 8months is as shown below.

No. of computers	250	200	220	270	220	260	300	240
Month	Jan	Feb	Mar	April	May	Jun	Jul	Aug

Calculate the four point moving averages for the data. (05marks)

8. Three forces of magnitudes 5N, 12N and 10N on bearings of 060^0, 210^0 and 330^0 respectively act on a particle. Find the resultant of the system of forces. (05marks)

SECTION B: (60 MARKS)

Attempt only **four** *questions in this section.*

9. The table below shows the cumulative frequency distribution of marks of 800 candidates who sat a national mathematics contest.

Mark(%)	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
F	30	80	180	330	480	610	700	760	790	800

a) Calculate the mean and standard deviation (08marks)
b) Construct an Ogive for the data and use it to estimate the;
i) Median mark (04marks)
ii) Quartile deviation (02marks)
c) Proportion of candidates that failed if the pass mark was 50% (01mark)

10. A quadratic curve has gradient function *(k - 2x)* and is such that when $x = 1$, $y = 2$ $andwehnx = {}^{-}1$, y = 0.
   (a) Find the value of **k** and state the equation of the curve. (07marks)
   (b) Sketch the curve. (05marks)

   (c) Find the area bounded by the curve and the x-axis. (03marks)

11. The table below gives marks obtained in mathematics examination (**M)** and physics Examination (**P)** obtained by 10 candidates.

Candidates	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
Math (M)	35	56	65	78	49	82	20	90	77	35
Physics (P)	57	75	62	75	53	100	38	82	82	20

(i) Draw a scatter diagram and comment. (07 marks)
(ii) Find the score in mathematics by a candidate who scored 82 in physics. (02marks)

(iii) Calculate the rank correlation coefficient and comment on your result. (06marks)

12. a) A and B are events such that P(A) = $^1/_3$,P(A or B but not both) = $^5/_{12}$

and P (B) = $^1/_4$. Calculate:

P (A∪B)
(04marks)
P (A′ ∩B)
(02marks)
P (B′/A) (02marks)

(b) Two men fire at a target. The probability that Allan hits the target is ½ and the probability that Bob does not hit the target is $^1/_3$. Allan fires at the target first followed by Bob. Find the probability that:

Both hit the target (02marks)
Only one hits the target (03marks)
None of them hits the target. (02marks)

13. a) Given that 2sin(A-B) = sin (A + B)
Show that tan A = 3tan B. (03marks)
Hence determine the possible values of A between – 180^0 and 180^0 when B=30^0.
(03marks)

(b) Solve the equation $sin2x - cos2x = 1$ for $0^0 \leq x \leq 360^0$.

(06marks)

(c) Without using tables or calculators, show that $\cos 75^0 = \frac{\sqrt{2}(\sqrt{3}-1).}{4}$
(03marks)

14. a) Bodies of mass 6kg and 2kg are connected by a light inextensible string passing over a smooth fixed pulley with the masses hanging vertically. Find the acceleration of the system when released from rest. (05marks)

(b) A body of mass 2kg moves along a smooth horizontal surface with speed of $2ms^{-1}$. It then meets a rough horizontal surface whose co-efficient of friction is 0.2. Find the horizontal distance it travels on the rough surface before it comes to rest.
(05marks)

(c) A particle of mass 5kg rests on a smooth surface of a plane inclined at angle 30^0 to the horizontal. When a force X acting up the plane is applied to the particle, it rests in equilibrium. Find the normal reaction and force X.
(05marks)

Uganda Advanced Certificate of Education
INTERNAL MOCK 2016
Subsidiary mathematics

INSTRUCTIONS:

- *Attempt **all** questions in section **A** and **any four** questions in section **B**.*
- *Time: 2 Hours 40 minutes*.

SECTION A:

1. A box contains 4 red and 7 black pens . Two pens are picked at random one ater the other without replacement. Find the probability that:

i) both pens are red.

ii) the pens are of a different colours.

2. i) Two mutually exclusive events A and B are such that

$$P(A) = \frac{2}{3} \quad and \quad P(B) = \frac{3}{7} \quad .$$

Find $P(A U 3)^1$.

ii) Two independent events Q and P are such that P(Q) = 0.5 and P(P) = 0.7 Find P(Q u P).

3. The following table gives the marks obtained by 10 students in maths and Economics.

Students	A	B	C	D	E	F	G	H	I	J
Maths(x)	74	48	71	66	60	47	72	80	40	48
Economics(Y)	50	44	38	41	48	45	57	50	47	36

Compute the rank correlation coefficient for the data and your results.

4. Given that a = 3i – 4j, b = -5i + 12j. Find

i) (3a + b).6

ii) angle between a and b.

5. Find the sum of the A.P series 11 + 13 + 15 + ……. + 89.

6. Differentiate the following with respect to x.

a) $(x + 5)^2 (x – 1)$

b) $\frac{1}{x^2} + x^2$

7. Given that $A = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 \\ -1 & 5 \end{pmatrix}$ $and \quad C = AB \quad ; Find \quad C^{-1}$

8. A body of mass 4kg decreases it's kinetic energy by 32J. If initially it had a speed of $5ms^{-1}$, find it's final speed.

SECTION B:

9. The quarter number of accidents in a certain town over a period of four years is given in the table below:

Year	**Q U A R T E R S**			
	1^{st}	2^{nd}	3^{rd}	4^{th}
2004	13	22	58	26
2005	16	28	61	25
2006	17	29	61	26
2007	18	30	65	29

i) Draw a graph to illustrate the data.
ii) Calculate the 4- quarterly moving averages and plot them on the same graph . Comment on your results.
iii) Estimate the accidents in the 1^{st} quarter of 2008.

10. A discrete random variable X has a probability density function.

$$P(X = x) = \begin{cases} \frac{a}{2x}, & x = 1, 2, ..., 4 \\ 0 & elsewhere \end{cases}$$

Determine the:

i) Expectation X.
ii) Variance and standard deviation of X.
iii) Probability of $P(X \leq 3)$.

11. The table below shows the prices and quantities of some four commodities A,B,C and D for the years 2011 and 2012.

Item	Price per unit(shs)		Quantities	
	2011	2012	2011	2012
A	100	120	36	42
B	110	100	69	88
C	50	65	10	12
D	80	85	11	10

Using 2011 as base year.

i) Calculate the prices index number for each item for 2012.
ii) Calculate the simple aggregate price index number for 2012.
iii) Calculate the weighted aggregate price index number fr 2012.
iv) Calculate value index number.

12. a) Given that matrices;

$$A = \begin{pmatrix} 5 & 1 \\ 0 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix} \quad and \quad C = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 5 & 2 \end{pmatrix}$$

Find :

i) ABC
ii) $(A + B)C$

b) By use of matrix method; Solve the simultaneous equations below; solve the simultaneous equations below:

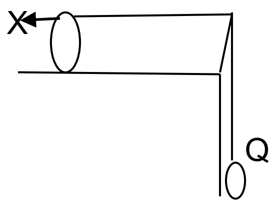
3x + 4y = 8
X + 2y = 3

13. a) Evaluate : $\int_0^6 2x\left(x^2 + 3\right)dx$

b) Determine the nature of turning points of the curve $y = 5 + 4x - x^2$; Hence sketch the curve.

c) Calculate the area bounded by the curve and the x – axis.

14. Take 9 =10ms^{-2}in this question. The diagram shows particle P of mass 0.5kg on a smooth horizontal table. P is connected to another particle Q, of mass 1.5kg by a taut light inextensible string which passes over a small fixed smooth pulley at the edge of the table , Q hanging vertically below the pulley. A horizontal force of magnitude X_N acts on P as shown.



i) Given that the system is in equilibrium find X.

II) Given that x = 12, find the distance travelled by Q in The first two seconds of its motion following the release of the system from rest. You may assume That P dies not react the pulley in this time.

END

<u>Section A (40 Marks)</u>

(Answer **all** questions in this section)

Qn 1: Given that $\frac{1}{\sqrt{2}} - \frac{\sqrt{2}+1}{1+3\sqrt{2}} = a\sqrt{2} + b$ where a and b are constants, find the values of a and b. [5]

Qn 2: The data below shows the marks obtained by 8 students in a certain test marked out of 30.

$$25, 20, 21, 23, 27, 23, 30, 23.$$

Find the:

(i). Mean mark,

(ii). Modal mark,

(iii). Median mark. [5]

Qn 3: If $3\log_t(10+3t) = 6$, find the value of t. [5]

Qn 4: A committee of 5 people is to be formed from a group of 6 men and 7 women.

(a). Find the number of possible committees. [2]

(b). What is the probability that there are only 2 women on the committee? [3]

Qn 5: Given the vectors $\underset{\sim}{\boldsymbol{a}} = \underset{\sim}{\boldsymbol{i}} - 2\underset{\sim}{\boldsymbol{j}}$, $\underset{\sim}{\boldsymbol{b}} = 3\underset{\sim}{\boldsymbol{i}} - \underset{\sim}{\boldsymbol{j}}$ and $\underset{\sim}{\boldsymbol{c}} = \underset{\sim}{\boldsymbol{i}} + 2\underset{\sim}{\boldsymbol{j}}$, find the length of the vector $5\underset{\sim}{\boldsymbol{a}} - \underset{\sim}{\boldsymbol{b}} + 3\underset{\sim}{\boldsymbol{c}}$. [5]

Qn 6: Given that $X \sim N(72, 225)$, find $\mathrm{P}(45 \leq X < 90)$. [5]

Qn 7: A and B are acute angles such that $\sin A = \frac{12}{13}$ and $\sin B = \frac{4}{5}$. Without the use of tables or a calculator, find the value of $\sin(A - B)$. [5]

Qn 8: Find the constant force necessary to accelerate a car of mass 1000 kg from 15 m s^{-1} to 20 m s^{-1} in 10 seconds against a resistance of 270 N. [5]

Section B (60 Marks)

(Answer only **four** questions from this section)

Question 9:

The table below shows the marks obtained by 10 students in a subsidiary mathematics and history exams.

History	80	75	65	90	95	98	78	65	54	60
Sub-Math	70	85	70	90	92	88	76	70	73	76

(a). (i). Represent the above data on a scatter diagram.

(ii). Draw the line of best fit.

(iii). If the student scored 75% in history, predict his score in subsidiary mathematics using your line of best fit.

(b). Calculate the spearman's rank correlation coefficient and comment on your result. [15]

Question 10:

A company that manufactures three types of radios requires diodes (D), valves (V), transistors (T) and capacitors (C). Sony requires 4 D, 3 V, 5 T and 2 C. Panasonic requires 4 V, 6 T and 1 C while Philips requires 2 D, 8 T and 5 C. The cost of each diode, valve, transistor and capacitor in thousands of shillings is 15, 5, 9 and 12 respectively.

(a). Write down:

(i). 3×4 matrix for the requirements of the radios.

(ii). 4×1 matrix for the cost of the accessories.

(iii). Use matrix multiplication to find the cost of manufacturing each radio.

(b). If Sony radio, Panasonic and Philips are sold at shs. 200,000; 150,000 and 160,000 each respectively. Use matrix method to find the percentage profit made by the company from sales of 20 Sony, 25 Panasonic and 15 Philips. [15]

Question 11:

(a). The table below shows the probability distribution function of a discrete random variable X.

X	-1	0	1	2
$P(X = x)$	0.4	0.1	0.3	0.2

(i). Calculate $P(X < 2/X > 0)$. [4]

(ii). Obtain the variance of X [4]

(b). The pdf of a continuous random variable X is given by

$$f(x) = \begin{cases} 3kx^2 & ; \quad 1 \le x \le 3, \\ 0 & ; \quad elsewhere. \end{cases}$$

Find:

(i). the value of k [3]

(ii). the expectation of X. [4]

Question 12:

A curve is such that $\frac{dy}{dx} = 3 - 2x$ and a point $P(1, 0)$ lies on the curve.

(a). Find the:

(i). equation of the curve. [3]

(ii). co-ordinates of the points where the curve meets the x-axis. [3]

(iii). co-ordinates and the nature of the stationary points. [4]

(b). Sketch the curve in (a) above and find the area enclosed by the curve and the x-axis. [5]

Question 13:

The table below shows the 3-point termly moving total scores of a student from S.1 over a period of three years.

Year	**Term one**	**Term two**	**Term three**
2014		1380	1215
2015	1020	915	870
2016	840	795	

(a). Calculate the 3-point moving averages for data.

(b). Represent the moving averages on a graph and comment on the trend of performance.

(c). Determine the moving totals in third term of 2016.

Question 14:

A train starts from station P and accelerates uniformly for 2 minutes reaching a speed of 72 km h^{-1}. It continues at this speed for 5 minutes and then is retarded uniformly for a further 3 minutes coming to rest at station Q. Draw a velocity-time graph and use it to find the:

(i). Time taken to cover the distance between P and Q in seconds.

(ii). Distance PQ in metres.

(iii). Average speed of the train.

(iv). Acceleration in m s^{-2}. [15]

*****END*****

Section A (40 Marks)

Answer **all** *the questions in this section.*

Qn 1: If $5\log_x(10+3x) = 10$, find the value of x. [5]

Qn 2: For a set of 10 numbers, $\sum x = 290$ and $\sum x^2 = 8469$. Find:

(a). the mean,

(b). the standard deviation. [5]

Qn 3: Given the polynomial $P(x) = 3x^3 + ax^2 + bx - 20$ where $P(1) = -14$ and $P(2) = 14$. Find the values of a and b. [5]

Qn 4: The table below shows the number of text books owned by 10 students of a certain class and their total marks in an exam.

Student	Number of text books	Total marks in an exam
A	5	290
B	8	370
C	2	184
D	9	366
E	7	277
F	5	190
G	3	385
H	10	200
I	1	281
J	4	331

Calculate the rank correlation coefficient between the number of text books and the total marks. Comment on your result at 5% level of significance. [5]

Qn 5: Given the vectors $\underset{\sim}{\boldsymbol{p}} = 3\underset{\sim}{\boldsymbol{i}} - 2\underset{\sim}{\boldsymbol{j}}$, $\underset{\sim}{\boldsymbol{q}} = 4\underset{\sim}{\boldsymbol{i}} + 2\underset{\sim}{\boldsymbol{j}}$ and $\underset{\sim}{\boldsymbol{r}} = \underset{\sim}{\boldsymbol{i}} + 2\underset{\sim}{\boldsymbol{j}}$, find the length of the vector $\underset{\sim}{\boldsymbol{p}} - 4\underset{\sim}{\boldsymbol{q}} + 3\underset{\sim}{\boldsymbol{r}}$. [5]

Qn 6: Three bags X, Y and Z, each contain black, red and blue pens as follows:

	Black pens	Red pens	Blue pens
Bag X	3	1	3
Bag Y	2	3	3
Bag Z	5	6	4

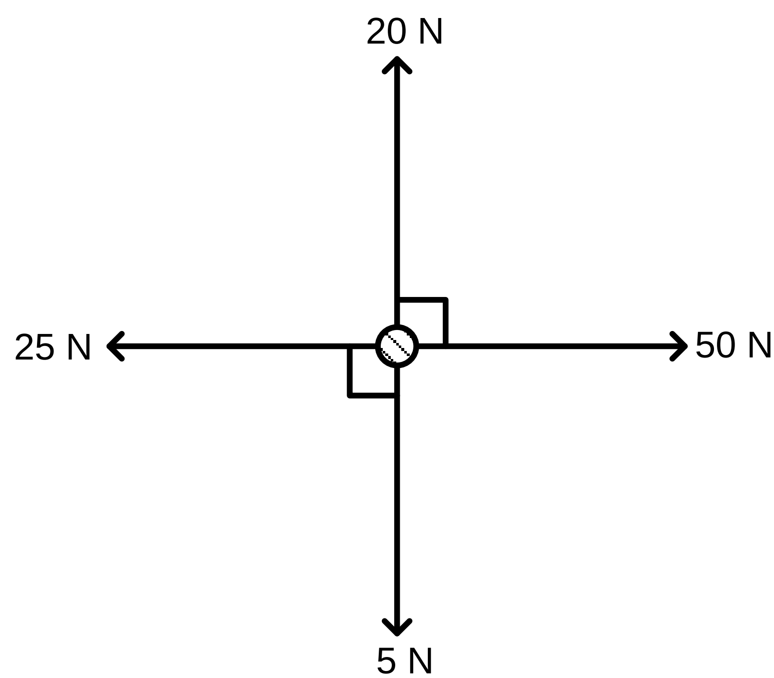
A bag is chosen at random and then a pen is randomly picked from the selected bag. Determine the probability that the pen picked is:

(i). a blue pen,

(ii). Not a blue pen. [5]

Qn 7: Given that $\sin\theta = \frac{-15}{17}$ and $180° \leq \theta \leq 360°$, find the value of $6\tan\theta + 8\,\text{cosec}\,\theta$. [5]

Qn 8: Find the magnitude and direction of the resultant force of the system of forces acting on a particle as shown below:



[5]

Section B (60 Marks)

Answer only **four** *questions from this section. All questions carry equal marks.*

Question 9:

The table below represents the length of leaves in millimetres (mm).

Length (mm)	Number of leaves
18.0 – 18.9	5
19.0 – 19.9	15
20.0 – 20.9	20
21.0 – 21.9	19
22.0 – 22.9	16
23.0 – 23.9	15
24.0 – 24.9	7
25.0 – 25.9	3

(a). Calculate the:
- (i). mean length.
- (ii). standard deviation. [6]

(b). Draw a cumulative frequency curve (ogive) and use it to estimate the:
- (i). median length.
- (ii). 80th percentile length.
- (iii). number of leaves whose length is below 22.45 mm. [9]

Question 10:

(a). Given that $P = \frac{1-\sin\theta}{1+\sin\theta}$,show that $P = (\sec\theta - \tan\theta)^2$. Hence deduce that if $\theta = 60°$, then $P = 7 - 4\sqrt{3}$. [8]

(b). Solve the equation $3\cos^2 x - 2\sin^2 x - \sin x + 1 = 0$ for $0° \leq x \leq 360°$. [7]

Question 11:

The cost of building a house is calculated from the price of cement, sand, bricks, roofing materials and labour. The table below gives the prices and price relatives of the items in the months of March and April respectively of 2019; and weights.

Item	March Unit Price (Ushs)	April Price Relatives (March = 100)	Weight
Cement	25,000	1.4	3
Sand	120,000	1.2	3
Bricks	230,000	0.65	2
Roofing materials	100,000	0.25	1
Labour	400,000	0.55	1

(a). Taking "Cement" as the base, calculate the price relatives for April.

(b). Determine the price of each item in April.

(c). Calculate the weighted aggregate price index for April using March as the base. Comment on your result. [15]

Question 12:

T is the tangent to the curve $y = x^2 + 6x - 4$ at $(1, 3)$ and N is the normal to the curve $y = x^2 - 6x + 18$ at $(4, 10)$. Find:

(a). the equation of the tangent T.

(b). the equation of the normal N.

(c). the coordinates of the point of intersection of T and N. [15]

Question 13:

The table below shows the monthly sales of a certain product in (shs "000") for the year 2018.

Month	Sales	Month	Sales
January	220	July	175
February	210	August	186
March	200	September	176
April	207	October	170
May	196	November	159
June	189	December	168

(a). Calculate 6-point moving totals and hence the moving averages. [6]

(b). (i). Plot on the same axes actual sales and moving averages. Comment on the trend of sales during the year.

(ii). Determine the sales in January 2019. [9]

Question 14:

(a). A boy pulls a box of mass 25 kg by means of a light inextensible string attached to it across a rough horizontal ground. The coefficient of friction between the box and the ground is 0.4. If the string is inclined at 30° to the horizontal and the box accelerates at 5 m s^{-2}, find the tension in the string. [7]

(b). A cyclist travels 100 m as he accelerates uniformly at a rate of Qm s^{-2} ; from a speed of 18 km h^{-1} to a speed of 36 km h^{-1}. Find:

(i). the value of Q.

(ii). the time taken to cover this distance. [8]

END

SECTION A: (40 MARKS)

Answer all questions from this section

1. A curve passing through point A (0, 8) has a gradient function $2x + 5$. Find the equation of the curve. *(05 marks)*

2. Solve the equation;
$3^{2x} - 3^{x} - 6 = 0.$ *(05 mark)*

3. The mass (kg) of 10 candidates in Naalya S.S was recorded as follows, 60, 83, 72, 51, 64, 80, 75, 56, 90 and 85. Find the standard deviation. *(05 marks)*

4. A discrete random variable x has the following probability distribution:
P(X = 1) = 0.1, P(X = 2) = 2P (X = 4) and P(X = 3) = 0.3. Find;
(i) P(X = 2)
(ii) Expected value of x *(05 marks)*

5. Find the reflex angle θ such that;
$2Sin^2\theta + Cos\theta + 1 = 0$ *(05 mark)*

6. Given that $\overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j}$ *and* $\overrightarrow{BA} = 7\mathbf{i} - \mathbf{j}$. Find;
(i) $\overrightarrow{OB}$
(ii) $\overrightarrow{OM}$, such that $\overrightarrow{AM} = ½ \overrightarrow{AB}$ *(05 marks)*

7. In 2014, the unit price of salt, price and cooking oil was 1600, 4200 and 3600 respectively. Given that the unit price in 2015 was P, 6800 and 3200 respectively and the simple aggregate price index was 125, find the value of P. *(05 marks)*

8. A car of mass 1.5 tonnes moves along a level road at a constant velocity of $80ms^{-1}$. If its engine exerts a driving force of 5kN, find the resistance that the car is experiencing. *(05 marks)*

SECTION B

9. The table below shows the termly expenditure in thousands of shillings by a school on National water;

Year	**Term I**	**Term II**	**Term III**
2013	303	324	318
2014	336	345	330
2015	321	300	312
2016	339	342	x

(a) Calculate the 3 – termly moving averages for the data. *(06 marks)*

(b) On the same axes, plot the graphs of the 3 termly moving averages and the termly expenditure. *(07 marks)*

(c) Use your graph to estimate the value of x. *(02 marks)*

10. (a) Sketch the curve;

$$y = x^2 - 2x - 3$$ *(10 marks)*

(b) Find the area bound by the curve and the x - axis. *(05 marks)*

11. A certain aptitude test has 10 statements that require a candidate to respond by writing true or false. A candidate passes if he or she scores at least eight questions correct;

(a) Find the probability that;

(i) A candidate gets exactly 5 questions correct

(ii) A candidate passes the test *(09 marks)*

(b) Calculate the expected number and standard deviation of the correctly answered questions. *(06 marks)*

12. The marks scored by candidates in a submaths exam were as follows;

64	74	78	59	67	55	61	54
80	58	76	58	74	65	63	83
72	60	71	52	61	57	68	69
62	73	64	59	62	53	81	68
50	75	67	53	80	77	60	71

(a) Construct a grouped frequency table for the data using equal classes of width 4 marks starting with 50 – 53 as the first class. *(02 marks)*

(b) State the;

(i) Median class

(ii) Modal class and its frequency. *(03 marks)*

(c) Calculate the;

(i) Mean mark

(ii) Standard deviation *(10 marks)*

13. (a) Given that $A = \begin{pmatrix} 2 & 4 \\ -6 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} x & 2 \\ y & 5 \end{pmatrix}$. Find the value of x and y such that AB = BA. Hence determine matrix p where P = AB = BA. *(08 marks)*

(b) Given the matrix $M = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}$. Find the value of λ for which matrix N = M - λ I is singular, where I is a 2 x 2 identity matrix. *(07 marks)*

14. Forces of 7N, 8N, 6N, 4N, 6N and 7N and 7N act along the sides of a regular hexagon ABCDEF in the directions AB, CB, CD, DE, EF and FA respectively. Find the magnitude and direction of the resultant force taking AB as the horizontal axis. *(15 marks)*

END

SMATA

2ND ANNUAL POST MOCK SEMINAR

S.6 Subsidiary Mathematics 2019

AT

ST. JOSEPH OF NAZARETH HIGH SCHOOL

5th October 2019

SEMINAR QUESTIONS

APPLIED MATH TOPICAL AREAS

Random variables

1. A random variable $\boldsymbol{x}$ has a probability density function given by

$$f(x) = \begin{cases} kx & 0 < \times < 3 \\ 3k \quad (4 - x) & 3 \leq \times \leq 4 \\ 0 \;\; elsewhere & \end{cases}$$

$\boldsymbol{k}$ is constant.

(i) Find the value of $\boldsymbol{k}$
(ii) $P(1 \leq \times \leq 3)$
(iii) $P(x \geq 2)$
(iv) $mean\ E(x)$
(v) variance $var(x)$
(vi) median

2. A random variable $\boldsymbol{x}$ has a probability density function given by

$$f(x) = \begin{cases} x^k & 0 \leq \times \leq 1 \\ 0 & elsewhere \end{cases}$$

(i) Find the value of $\boldsymbol{k}$
(ii) $P(x > 0.5)$.

3. A variable $\boldsymbol{x}$ has a probability distribution shown below

x	1	2	5	10
$f(x)$	0.5	P	0.12	q

Given that the mean $\boldsymbol{E(x) = 2.5.}$ Find the values of p and q.
Hence find the mode and median

4. A random variable x has a pdf given by $f(x)\begin{cases} \boldsymbol{cx} & \boldsymbol{x = 1, 2, 3, 4} \\ \boldsymbol{c\ (8-x)} & \boldsymbol{x = 5, 6, 7} \\ \boldsymbol{0} & \boldsymbol{otherwise} \end{cases}$

Find the

(i) value of c
(ii) P(3< $x \leq 5$)
(iii)P(x> $2/x \leq 6$)

Binomial and normal distribution

5. a) Given that $x{\sim}B(10, P)$ and variance $\frac{15}{8}$. Determine the possible of P hence find $P(x \geq 7)$ if p is less than 0.5

b) A certain type of cabbage has a mass which is normally distributed with mean $\boldsymbol{1kg}$ and standard deviation $\boldsymbol{0.15kg}$. In a lorry loaded with these cabbages. Find the probability that a cabbage weighs
(i) greater than $\boldsymbol{0.79kg}$
(ii) less than $\boldsymbol{1.13kg}$
(iii)between $\boldsymbol{0.85kg}$ and $\boldsymbol{1.15kg}$
(iv)number of cabbages between $\boldsymbol{0.75kg}$ and $\boldsymbol{1.29kg}$ if $\boldsymbol{10,000\ cabbages}$ were loaded.

Probability

6. At a certain wedding there are $\boldsymbol{5\ light}$ and $\boldsymbol{4\ dark}$ – skinned groomsmen. If $\boldsymbol{7\ groomsmen}$ are selected at random. What is the probability that at least **4** are light skinned?

7. a) A and B are events such that $\boldsymbol{P(A)} = {}^{\boldsymbol{5}}/_{\boldsymbol{12}}\ \boldsymbol{P(A \cap B)} = {}^{\boldsymbol{1}}/_{\boldsymbol{4}}$ and $\boldsymbol{P(}{}^{\boldsymbol{A}}/_{\boldsymbol{B}}\boldsymbol{)} = {}^{\boldsymbol{3}}/_{\boldsymbol{4}}$ find $\boldsymbol{P(A \cup B)}$
b) Given that $\boldsymbol{P(A) = 4x,\ P(B)} = {}^{\boldsymbol{1}}/_{\boldsymbol{3}}\ \boldsymbol{P(A \cap B) = x}$ and $\boldsymbol{P(A \cup B) = 8x}$. Find the value of $\boldsymbol{x}$ and $\boldsymbol{P(A^1 \cap B)}$.

8. A box has 5 green and 6 white sweets, three sweets are pick at random one after other without replacement. Find the probability that.

(i) All sweets are of the same colour
(ii) Sweets are of different colours
(iii)The third sweet is green.

Statistics

8. The table below shows the marks scored by 100 students in a Math test

$Marks$	$20\ -$	$30\ -$	$40-$	$50-$	$60-$	$70-$	$80-$	$90\text{-} < 100$
f	5	15	20	19	16	15	7	3

a) Construct a frequency distribution table and use it to calculate;
   (i) Mean mark
   (ii) Variance and standard deviation

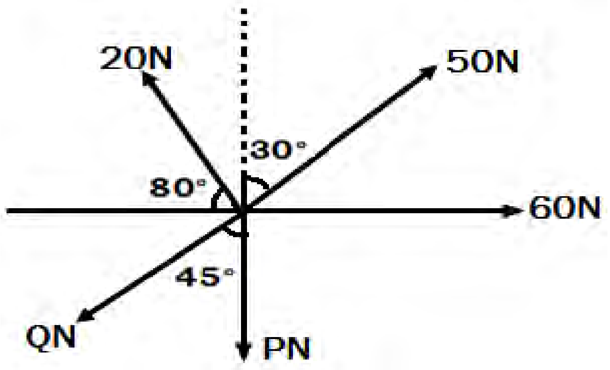
b) Draw an ogive and use it to estimate
   (i) Median mark
   (ii) Pass mark if 60 students passed
   (iii) Compute the modal mark

9. The time taken by Submaths students to complete an exercise was recorded and the following results were obtained. $\mathbf{32.5, 34.5, 33.5, 29.8, 30.9}$, and $\mathbf{31.8}$

Calculate;
   (i) Mean time
   (ii) Variance

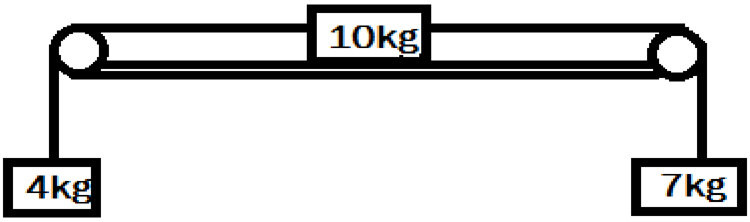
Mechanics

10. Five forces act as shown below and are in equilibrium. Find the forces $\boldsymbol{P}$ and hence find the magnitude and direction of the resultant force.



11. a) Two particles $\boldsymbol{A}$ and $\boldsymbol{B}$ of masses $\mathbf{3}\boldsymbol{kg}$ and $\mathbf{5}\boldsymbol{kg}$ are connected by a light string passing over a smooth fixed pulley. Find
   (i) their common acceleration
   (ii) the tension in the string

12. Three bodies are connected by a light inextensible strings passing over a smooth pulley. A mass of $\mathbf{10}\boldsymbol{kg}$ lies on a smooth surface of a table with the $\mathbf{4}\boldsymbol{kg}$ and $\mathbf{7}\boldsymbol{kg}$ masses hanging freely as shown below.



If the system is released from rest. Find the;

(i) Acceleration and tension in the string

(ii) Distance moved by the $7kg$ mass after $2\ seconds$.

13. A car travels along a straight road. The car starts at A from rest and accelerates for $30\ seconds$ at a constant rate until it reaches a speed of $25ms^{-1}$. The car continues at $25ms^{-1}$ for $T\ seconds$, after which it decelerates for $10\ seconds$ until it reaches a speed of $15ms^{-1}$ as it passes B. The distance AB is $80km$.
    a) Sketch the velocity – time graph for the journey between A and B.
    b) Find the value of T. Hence, total time of the journey from A to B.

Moving average and scatter diagram

14. The table below shows quarterly income in ($millions\ of\ shillings$) of a certain company over a period of three years.

Quarters	Years		
	2006	**2007**	**2008**
1	70	80	120
2	85	105	115
3	140	150	170
4	60	75	95

a) Calculate a four point moving average for the above data.
b) On the same axes, plot the graphs of original data and four point moving averages.
c) Comment on the results in (a) above and use the graph to estimate the expected revenue in the
   (i) First quarter of 2008
   (ii) Fourth quarter of 2008

15. The table below shows the three point moving totals for a number of tourists who visited a certain National Park over a period of four years.

Year	Round 1	Rounds 2	Rounds 3
2015		1530	1310
2016	1300	1290	1205
2017	1194	1110	1096
2018	1052	1014	

a) Calculate the three point moving averages for the above data.
b) Represent the moving averages on the same axes and comment on the trend of visitors visiting the National Park.
c) Use the graph to estimate the totals in $Round\ 3$ of 2018.

16. Two adjudicators at a Musical competition awarded marks to ten Pianist as follows

Pianist	A	B	C	D	E	F	G	H	I	J
Adjudicator x	78	66	73	73	84	66	89	84	67	77
Adjudicator y	81	68	81	75	80	67	85	83	66	78

a) Draw a scatter diagram for the data and comment on your graph.
b) Calculate the spearman's rank correlation coefficient and comment at 5% level of significance.

Price indices

17. In 1996, the prices of pens, pencils and books were $shs. 200$, 50 and 400 respectively. Their respective prices in 2006 were $shs. 500, 200$ and P. Given the simple aggregate price using 1996 as the base year was 136. Find the value of P.

18. The cost of making toasted bread is calculated from the cost of baking flour, sugar, milk, eggs and food colour.

Item	2017	2018	Weight
Wheat	5100	5700	10
Sugar	4000	3600	5
Milk	1000	1400	4
Eggs	9000	8500	2
Food colour	1000	1300	1

a) Using 2017 as the base year, calculate
   (i) The price relatives for each item.
   (ii) Simple aggregate price index.
   (iii) Average price.
   (iv) Cost of living index and comment
   (v) The weighted aggregate price index.

b) Using the index in (ii) estimate the cost of making bread in 2017 if the cost in 2018 was $2,200$.

~END

PURE MATH QUESTIONS

1. Solve the simultaneous equations below

$$2^{x+3} + 3^{y+2} = 3^2$$
$$2^x + 3^y = 4$$

2. A polynomial $P(x) = x^4 + ax^3 + bx^2 + 5x + 3$ has a remainder of $2x + 1$ when divided by $x^2 + 3x + 2$. Find the values of a and b.

3. Given that $\log(3x + 8) - 3\log 2 = \log(x - 4)$. Find the value of x.

4. Solve for x in $\cos 2x + 5\cos x = 2$ for $0° < \times < 360°$

5. Given that vectors $a = 2\hat{\imath} - 4\hat{\jmath}$ and $b = 3\hat{\imath} + 5\hat{\jmath}$ find the modulus of the vector $5a + 2b$ and the angle between vector **a** and **b**

6. When $H(x) = x^3 - ax^2 - bx + 24$ is divided by $x + 1$, the remainder is 19 and $x + 2$ is a factor of $H(x)$. Find a and b hence factorise.

7. If $\propto$ and β are roots of the equation $2x^2 + 3x + 5 = 0$. Form an equation whose roots are $\frac{\beta}{\propto - 4}$ and $\frac{\propto}{\beta - 4}$

8. The sum of the first $12\ terms$ of an arithmetic progression AP is 120. The eighth term is four times the sum of the fourth and fifth terms. Determine the;
   (i) First term and common difference of AP.
   (ii) Sum of the first $20\ terms$

9. Solve for n if $\frac{8^{n+2} \times 4^{2n-1}}{2^n \times 4^{\frac{n}{2}}} = 16$

10. Express $\frac{3\sqrt{2} - 2\sqrt{3}}{3\sqrt{2} + 2\sqrt{3}}$ in the form $a + b\sqrt{c}$ and hence state the values of a, b and c.

11. Given that $\cos A = {}^4/_5$ and $sinB = {}^{12}/_{13}$ $where\ A\ ans\ b\ are\ acute$. find
   (i) $\sin A + \tan A$
   (ii) $\sin(A - B)$

12. Given that $A = \begin{pmatrix} 5 & 1 \\ 4 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$. Find A^{-1} and AB.

13. A cake is made up of ingredients, flour oil, eggs and sugar. A sample of three cakes made by a certain bakery was found to have been made form:
   Cake A: $2\ kg\ of\ flour$, $1\ litre\ of\ oil$, $8\ eggs$ and $250g\ of\ sugar$
   Cake B: $3kg\ of\ flour$, $2\ litres\ of\ oil$, $12\ eggs$ and $500g\ of\ sugar$
   Cake C: $5kg\ of\ flour$, $2.5litres\ of\ oil$, $20\ eggs$ and $750g\ of\ sugar$.

In **2017**, the cost of $\boldsymbol{a\ kg\ of\ flour}$, $\boldsymbol{a\ litre\ of\ oil}$, $\boldsymbol{an\ egg}$ and $\boldsymbol{kg\ of\ sugar}$ were at $\boldsymbol{shs.2,200}$, $\boldsymbol{shs.\ 2,500}$, $\boldsymbol{shs,400}$ and $\boldsymbol{shs.3,200}$ respectively.

In **2018,** the cost of $\boldsymbol{a\ kg\ of\ flour}$, $\boldsymbol{a\ litre\ of\ oil}$, $\boldsymbol{an\ egg}$ and $\boldsymbol{a\ kg\ of\ sugar}$ were $\boldsymbol{shs.3,000}$, $\boldsymbol{shs.\ 3,000}$, $\boldsymbol{shs.500}$ and $\boldsymbol{shs.3,600}$ respectively.

(i) Write down a $\boldsymbol{3 \times 4\ matrix}$ to represent the ingredients items of the cakes and $\boldsymbol{4 \times 1}$ **matrix** for the costs of the items in the two years.
(ii) Using matrix multiplication, calculate the total cost of making the three cakes for each years **2017** and **2018.**
(iii) What is the difference between the costs in the two years?

14. The gradient function of a curve at a point $(\mathbf{0}, \mathbf{13})$ lies on the curve.
    (i) Find the equation of the curve.
    (ii) Determine the coordinates and nature of its turning points.
    (iii) Sketch the curve hence find the area between the curve and the line $\boldsymbol{y = 13}$

15. Find the equation of the tangent to the curve $\boldsymbol{x^2y = 2 + x}$ at a point when $\boldsymbol{x = -1}$

16. $\boldsymbol{T}$ is a tangent to the curve $\boldsymbol{y = x^2 + 6x - 4}$ at $(\mathbf{1}, \mathbf{3})$ and $\boldsymbol{N}$ is the normal to the curve $\boldsymbol{y = x^2 - 6x + 18}$ at $(\mathbf{4}, \mathbf{10})$**.** Find
    (i) Equation of the tangent $\boldsymbol{T}$
    (ii) Equation of the normal $\boldsymbol{N}$
    (iii) Coordinates of the point of intersection of $\boldsymbol{T}$ and $\boldsymbol{N}$.

17. The points $\boldsymbol{P}$ and $\boldsymbol{Q}$ have position vectors $\boldsymbol{OP = -2\hat{\imath} - 5\hat{\jmath}}$ and $\boldsymbol{OQ = \hat{\imath} - 2\hat{\jmath}}$ respectively. $\boldsymbol{R}$ is a point such that $\boldsymbol{OR = OP + \lambda\ PQ}$.
    a) Find the
        (i) Value of $\boldsymbol{OP, OQ}$
        (ii) Angle between $\boldsymbol{OP}$ and $\boldsymbol{OQ}$

    b) Determine the vector
        (i) $\boldsymbol{PQ}$
        (ii) $\boldsymbol{OR}$ in terms of λ
        (iii) Value of λ for which $\boldsymbol{OR}$ is perpendicular to $\boldsymbol{PQ}$.

18. The rate of delay of a radioactive material is proportional to the amount $\boldsymbol{x}$ grams of the material present at any time $\boldsymbol{t}$. initially, there was $\boldsymbol{100g}$ of material. After $\boldsymbol{5\ minutes}$, the material had reduced to $\boldsymbol{90g}$.
    a) Form – differential equation for the rate of delay of the material.
    b) Solve the differential equation in (a) above.
    c) Find the;
        (i) Amount of the radioactive material present after $\boldsymbol{20\ minutes}$
        (ii) Time taken for the material to reduce to $\boldsymbol{20g}$**.**

AIM AT GETTING A POINT AT UNEB

SECTION A

1. Given that $\frac{1}{\sqrt{2}} - \frac{\sqrt{2}+1}{1+3\sqrt{2}} = a\sqrt{2} + b$ where a and b are constants, find the values of a and b. ***(5marks)***

2. Events A, B and C are such that $(A) = \frac{2}{7}$, $P(B) = \frac{3}{8}$ and $P(C) = \frac{3}{5}$. Given that A and C are independent events and B and C are mutually exclusive events. Find ;
   (i.) $P(AUC)$
   (ii.) $P(BUC)'$ ***(5marks)***

3. The table shows two variables x and y

X	60	5	30	75	45	36	50	20	10	40
y	38	66	44	28	47	56	48	56	62	54

   Calculate the rank correlation coefficient between x and y and comment on your results. ***(5marks)***

4. Given that $\boldsymbol{a} = 2\boldsymbol{i} + j$, $\boldsymbol{b} = 3\boldsymbol{i} - 4\boldsymbol{j}$ and $\boldsymbol{c} = 2\boldsymbol{a} + \boldsymbol{b}$ **.** Find;
   (i.) Vector **C**
   (ii.) Modulus of vector **c**. ***(5marks)***

5. Solve the equation $2sin2\theta = 3cos\theta\ , for\ 0^o \leq \theta \leq 360^o$. ***(5marks)***

6. The cost of building a house is calculated from the costs of cement, sand, bricks, roofing materials and labour.

   The table below gives the costs of these items in 2010 and 2014.

ITEM	**COST (shs.) PER UNIT 2010**	**COST (shs.) PER UNIT 2014**	**WEIGHTS**
Cement	25000	30000	20
Sand	120000	125000	3
Bricks	230000	290000	2
Roofing materials	600000	800000	1
Labour	400000	450000	1

   Using 2010 as base year , calculate the weighted aggregated price index for 2014. And comment on the result. ***(5marks)***

7. Solve the equation $3(1-x)^2 + 7(1-x) + 4 = 0$. ***(5marks)***

8. A car decelerating at $0.95ms^{-1}$passes a certain point with a speed of $30ms^{-1}$. Find its velocity after 10s and the distance covered in that time. ***(5marks)***

SECTION B

9. (a.) The data below shows the expenditure in thousands of shillings of 8 families during the month of May 2015.

25, 20, 21, 23, 27, 23, 30, y.

If the mean expenditure was shs.24,000. Find the;

(i.) Value of y

(ii.) Modal expenditure

(iii.) Median expenditure. ***(6marks)***

(b.)The total score in Mathematics and English obtained by 30 students were

164	148	138	145	153	157	147	174	143	148
136	154	170	128	140	178	165	121	149	161
143	150	139	151	139	165	151	133	145	126

Copy and complete the frequency distribution table below for the above data

class	**tallies**	**Frequency (f)**	**Class mark (x)**	**fx**	**Cumulative frequency**	**Class Boundaries**
120 – 129						
130 – 139						
140 – 149						
150 – 159						
160 – 169						
170 – 179						
		Ɛf = ...		**Ɛfx = ...**		

Calculate

(i.) The mean mark

(ii.) The median mark. ***(9marks)***

10. The following table shows the milk production in litres of a diary farm in each quarter of the years 2012, 2013, and 2014

year	**1st quarter**	**2nd quarter**	**3rd quarter**	**4th quarter**
2012	2490	3640	5015	1480
2013	2360	3520	4890	1375
2014	2155	3265	4630	1170

(a.)Calculate the four point moving averages. ***(6marks)***

(b.) (i) On the same axes, plot and draw graphs of the original data and the four – point moving averages.

(ii.) Comment on the trend of milk production. ***(6marks)***

(c.)Use your graph in (b) to estimate the amount of milk produced by the farm in the 1st quarter of 2015. ***(3marks)***

11. (a.) The probability mass function of s random variable X is given by

$$P(X=x)=\begin{cases} kx+\frac{1}{5} & ,x=-2,-1,0,1,2. \\ 0 \quad , & elsewhere \end{cases}$$

Where k is a constant.

(i.) Determine the value of k

(ii.) Compute the mean

(iii.) Find the mode

(iv.) Find $P(x<0)$. ***(15marks)***

12. (a.) Given that $V=7+5t-3t^2$, find the maximum value of V.

(4marks)

(b.) If $\frac{dy}{dx}=7$, and $x=1, y=10,\ find;$

(i.) y in terms of x

(ii.) x when y = 24 ***(7marks)***

(c.) Evaluate $\int_2^5\left(\frac{x^2}{3}\right)dx$ ***(4marks)***

13. (a.) Use matrix to solve the simultaneous equations

$$8x - y = 6$$
$$3x + 2y = 26$$
(6marks)

(b.) Mukisa ordered for the following items from a shop; 2 kg of sugar, 3 bars of soap and 1 packet of tea leaves. And Okello ordered for 4 kg of sugar, 1 bar of soap and 2 packets of tea leaves. If the cost of sugar is shs. 2500 per kg, soap shs.2100 a bar and tea leaves shs. 1200 a packet.

(i) Form a matrix of order 2 X 3 for the items ordered

(ii) for a matrix of order 3 X 1 for the costs of the items.

(iii) Use matrix multiplication to find the bills paid by each person.

(9marks)

14. (a.) Two points A and B are 2cm apart, if a force of 50N is used to move a body from A to B in 15 s.

(i.) Find the work done.

(ii.) Calculate the rate at which the force is acting. ***(4marks)***

(b.) The point P is 5 metres vertically above point O. A body of mass 0.6 kg is projected from

P vertically downwards with a speed of $3.5ms^{-1}$. Find the speed of the body when it reaches O. ***(11marks)***

End -

(05 marks)

2. The table below shows cost per kg of some items commonly used by a certain family.

Item	Beans	Posho	Salt	G. nuts	Rice
Cost per kg	3000	2000	500	2600	2200

Using **posho** as the **baseprice**, calculate the **cost of living** index and comment on your results. (05 marks)

3. Solve the equation $\mathbf{2\,sec^2\,\theta - 3 + \tan\theta = 0}$ for values of $\boldsymbol{\theta}$ from **0**° to $\mathbf{360}$° (05 marks)

4. $\boldsymbol{A}$ and $\boldsymbol{B}$ are two independent events such that $\boldsymbol{p\,(A) = 0.3}$ and $\boldsymbol{p(B) = 0.35}$ evaluate;
   (i) $\mathbf{p\,(A \cap B)}$
   (ii) $\mathbf{p\,(A \cup B)}$
   (iii) $\mathbf{p\,(^{A}/_{B})}$

(05 marks)

5. Solve the differential equation $\frac{dy}{dx} = \frac{7x^2+1}{8y}$; given that $\boldsymbol{y}$ **= 2** when $\boldsymbol{x = 0}$

(05 marks)

6. The table below shows the weight of students in a certain class.

Weight (kg)	5 -	10 -	15 -	20 -	25 -	30 -	35 - 40
Cumulative frequency	2	7	15	30	35	38	40

Calculate the variance for the data. (05 marks)

7. The third term of a geometrical progression **(GP**) is **10** and the **sixth** term is **80**. Find the sum of the first six terms. (05 marks)

8. $\boldsymbol{PQRS}$is a square of side "$\boldsymbol{a}$"**.** Forces of magnitude $\mathbf{2N, 1N, \sqrt{2N}}$; and $\mathbf{4N}$ act along $\boldsymbol{PQ, QR, PR}$and$\boldsymbol{SP}$respectively. Thedirection being in the order of letters. Find the magnitude and direction of the resultant force. (05 marks)

SECTION B (60 MARKS)

(Attempt **any four** questions**)**

9. The number of customers who visit a certain bank for the days **Monday** to **Friday** were recorded for **three** weeks.

Week	Mon	Tue	Wed	Thur	Fri
I	142	177	213	171	138
II	125	172	191	170	131
III	114	158	192	155	127

(a) Calculate the **Five – point** moving averages for the data. (06 marks)

(b) (i) On the same axes; plot the original data and the five – point moving averages. (05 marks)

(ii) Comment on the trend of the number of customers who visit the bank over the three weeks. (01 mark)

(iii) Use your graph to estimate the number of customers who will visit the bank on **Monday** in the **Four (IV)**week. (03 marks)

10. The table below shows the percentage preference of nine most popular holiday destinations as sampled by a tour company for two years **2015** and **2016**

Holiday destination	A	B	C	D	E	F	G	H	I
2015 (x)	90	80	78	78	50	40	30	20	10
2016 (x)	79	90	80	60	60	35	30	60	22

(a) (i) Draw a scatter diagram for the data and comment on the correlation between **x** and **y**.

(ii) Draw a line of best fit on your scatter diagram

(iii) Use the line of best fit to find the value of $\boldsymbol{y}$ when $\boldsymbol{x = 45}$ (08 marks)

(b) Calculate the **rank correlation** co – efficient. (07 marks)

11. The points $\boldsymbol{p, Q}$and $\boldsymbol{R}$ have position vectors $\mathbf{2\imath + 2j}$; $\mathbf{\imath + 6j}$ and $\mathbf{-7\imath + 4j}$ respectively.

(a) (i) Find the vectors $\boldsymbol{QR}$ and $\boldsymbol{PQ}$

(ii) Show that triangle $\boldsymbol{PQR}$ is right – angled at $\boldsymbol{Q}$.(07 marks)

(b) Find the angle between $\boldsymbol{PR}$ and $\boldsymbol{PQ}$. (08 marks)

12. A sugar factory sells sugar in bags of **mean** weight **50kg** and **variance 6.25kg**.Given that the weights of the bags are normally distributed;

(a) Find the probability that the weight of any bag of sugar randomly selected lies between **51.5kg** and **53 kg**.

(b) Calculate the percentage of bags whose weights;

(i) exceed **54 kg**

(ii) lies between **46.58 kg** and **55.58 kg**

(c) Determine the number of bags that will be rejected out **1000 bags** purchased for weighing below **45kg**.

(15 marks)

13. (a) Sketch the curve $\boldsymbol{y = x^2 + 2x - 24.}$ (10 marks)

(b) Find the area enclosed by the curve and the $\boldsymbol{x}$ −axis from $\boldsymbol{x = -4}$ to $\boldsymbol{x = 4}$. (05 marks)

14. A motorist sets of from **townA** and accelerates uniformly for $\mathbf{T_1}$ seconds covering a distance of **500m**. He then travels at a speed of **V km/hr** for $\mathbf{T_2}$ seconds covering a further distance of **1000m**. He then decelerates uniformly for $\mathbf{T_3}$ seconds coming to rest at**townB**. If the total time taken is **5minutes** and that $\boldsymbol{T_1 = {}^1/_2\, T_3}$;

(a) Sketch a velocity – time graph.

Find; $\mathbf{T_1}$**,** $\mathbf{T_2}$**,** $\mathbf{T_3}$**, V** and distance **AB".** (15 marks)

~END~

SUCCESS IS A STRUGGLE!

SECTION A

1. When expression $x^5 + 4x^2 + ax + b$ is divided by $(x^2 - 1)$, the remainder is $(2x + 3)$, find a and b.

2. Solve for θ: $\tan\theta + 2 = 2\sec^2\theta$, for $0^o \le \theta \le 360^o$.

3. The events A and B are independent. If $P(A) = 0.3$ and $P(B) = 0.5$, find
(a) $P(A \cup B)$ b) $P(A \cap B')$

4. X is a random variable such that $P(X = x) = \begin{cases} \frac{1}{10}x & ; \quad x = 1,\ 2,\ 3 \\ \text{c} & ; \quad \text{x} = 4,\ 5 \\ 0 & ; \quad \text{elsewhere} \end{cases}$

Find (i) the value of the constant c (ii) E(X)

5. A box contains 6 blue pens and 10 red pens. Two pens are picked one after with replacement, find the probability that both are of the same colour.

6. Differentiate w.r.t x: i) $y = (x+1)(2x-1)$ ii) $y = \frac{(x^2+1)(x^2+6)}{x^2}$

7. Given that $\mathbf{u} = 4\sqrt{2}(\cos 135^o \mathbf{i} + \sin 135^o \mathbf{j})$ and $\mathbf{v} = 6(\cos 270^o \mathbf{i} + \sin 270^o \mathbf{j})$, find $|\mathbf{u} + \mathbf{v}|$.

8. Solve the equation: $2x^3 - x^2 - 5x - 2 = 0$.

SECTION B

(Attempt any **FOUR** questions from this section)

9. Below are the marks obtained by students in Sub Mathematics at Rubaga Girls Senior Secondary school.

Class	$40 - < 50$	$50 - < 60$	$60 - < 70$	$70 - < 80$	$80 - < 90$	$90 - < 100$
Frequency	5	8	12	18	4	3

a) Calculate: i) the modal ii) the mean mark

b) Draw an ogive curve and use the graph to estimate:

i) the median ii) 60^{th} percentile

iii) Number of students who scored below 75%.

10a) A random variable X has a probability density function given by

x	0	1	2	3
$P(X = x)$	$\frac{1}{10}$	$\frac{3}{10}$	b	$\frac{2}{10}$

i) Determine the value of b.

ii) Calculate the variance.

b) The following scores obtained by students who competed in the swimming and dancing competitions in 2014.

Swimming(Y)	600	150	375	550	200	450	250	300	525	350
Dancing(X)	110	170	140	115	165	130	155	150	120	145

i) Draw a scatter diagram and use your graph to estimate:

a) a score obtained by a dancer who scored 425 in swimming.

b) a score obtained by a swimmer who scored 180 in dancing.

ii) Calculate the rank correlation coefficient and make a comment.

11a) The table below shows the prices of four commodities and their weights in 2006 and 2007.

Commodity	Price(**U shs**)		**Weight**
	2006	**2007**	
Banana(1 bunch)	3000	8000	4
Meat(1kg)	2500	3000	3
Milk(1 litre)	300	400	2
Sugar(1 kg)	1500	1800	1

Taking 2006 as the base year, find:

i) the price relative for each commodity.
ii) Weighted price index for all the commodities.

b) The average prices of a bunch of a kilo of sugar in each quarter of a year are given in the table below.

	1st	2nd	3rd	4th
1998	4500	5000	5200	5500
1999	5500	5700	6000	6400
2000	6200	6500	6800	7200
2001	7400	Y		

a) Calculate the 5 point moving averages.

b) On the same graph, show the raw data and the 5 point moving averages and hence, comment on the trend of the prices for this period and use your graph to estimate the value of Y.

12a) Evaluate: i) $\int_1^2 3x^2 - 4x + 2\, dx$ ii) $\int_1^4 \sqrt{x} - \frac{4}{\sqrt{x}} + 2\, dx$

b) Determine the turning points of the curve $y = 2x^2 + 5x - 3$ and sketch the curve $y = 2x^2 + 5x - 3$ and hence, find the area enclosed by the curve and the x axis.

13a) The second term of an Arithmetic progression is 15 and the fifth term is 21. Find the first term and the common difference and hence find the sum of the first 20 terms of the Arithmetic progression.

b) Given that $A = \begin{pmatrix} 1 & 3 \\ 2 & -2 \end{pmatrix}$, evaluate $\det(A^2 - 2A)$.

c) Use the matrix method to solve the simultaneous equations. $\begin{matrix} 2x + y = 3 \\ 3x + 5y = 5 \end{matrix}$

14i) A car initially moving at a speed of $80m\,s^{-1}$ decelerates uniformly and attains a velocity of $40m\,s^{-1}$ for $20s$ and comes to rest in the next $30s$. Sketch a velocity – time graph and use it to calculate the average velocity.

ii) Forces of magnitude $2\sqrt{2}N$, $4N$ and $6N$, act on a body at angles 45^o, 240^o, 330^o with the positive $x-$axis. Draw a clear force diagram and find the resultant force.

END

UGANDA ADVANCED CERTIFICATE OF EDUCATION

MOCK EXAMINATIONS, 2016

SUB-MATHEMATICS

S475/1

INSTRUCTIONS TO CANDIDATES

Answer **ALL** questions from section A and **FOUR** questions from section B

Show all the working clearly

Scientific non-programmable calculators and mathematical tables with a list of formulae may be used

where necessary use g = 9.8ms^{-1}

SECTION A

1. Given that $\frac{1}{\sqrt{2}} - \frac{\sqrt{2}+1}{1+3\sqrt{2}} = a\sqrt{2} + b$ where a and b are constants , find the values of a and b.

(5marks)

2. Events A, B and C are such that $(A) = \frac{2}{7}$, $P(B) = \frac{3}{8}$ and $P(C) = \frac{3}{5}$. Given that A and C are independent events and B and C are mutually exclusive events. Find ;

   (i.) $P(AUC)$

   (ii.) $P(BUC)'$ ***(5marks)***

3. The table shows two variables x and y

X	60	5	30	75	45	36	50	20	10	40
y	38	66	44	28	47	56	48	56	62	54

Calculate the rank correlation coefficient between x and y and comment on your results.

(5marks)

4. Given that $\boldsymbol{a} = 2\boldsymbol{i} + j$ and $\boldsymbol{b} = 3\boldsymbol{i} - 4\boldsymbol{j}$ and $\boldsymbol{c} = 2\boldsymbol{a} + \boldsymbol{b}$ **.** Find;
   (i.) Vector **C**
   (ii.) Modulus of vector **c**. ***(5marks)***
5. Solve the equation $2sin2\theta = 3cos\theta\ , for\ 0^o \leq \theta \leq 360^o$.

(5marks)

6. The cost of building a house is calculated from the costs of cement, sand, bricks, roofing materials and labour.

   The table below gives the costs of these items in 2010 and 2014.

ITEM	COST (shs.) PER UNIT 2010	COST(shs.) PER UNIT 2014	WEIGHTS
Cement	25000	30000	20
Sand	120000	125000	3
Bricks	230000	290000	2
Roofing materials	600000	800000	1
Labour	400000	450000	1

Using 2010 as base year , calculate the weighted aggregated price index for 2014. And comment on the result. ***(5marks)***

7. Solve the equation $3(1-x)^2 + 7(1-x) + 4 = 0$.

(5marks)

8. A car decelerating at 0.95ms^{-1}passes a certain point with a speed of 30ms^{-1}. Find its velocity after 10s and the distance covered in that time. ***(5marks)***

SECTION B

9. (a.) The data below shows the expenditure in thousands of shillings of 8 families during the month of May 2015.

25, 20, 21, 23, 27, 23, 30, y.

If the mean expenditure was shs.24,000. Find the;

(i.) Value of y

(ii.) Modal expenditure

(iii.) Median expenditure. ***(6marks)***

(b.)The total score in Mathematics and English obtained by 30 students were

164	148	138	145	153	157	147	174	143	148
136	154	170	128	140	178	165	121	149	161
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Copy and complete the frequency distribution table below for the above data

class	**tallies**	**Frequency (f)**	**Class mark (x)**	**fx**	**Cumulative frequency**	**Class Boundaries**
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130 – 139						
140 – 149						
150 – 159						
160 – 169						
170 – 179						
		Ɛf = …………		**Ɛfx = ……….**		

Calculate

(i.) The mean mark

(ii.) The median mark. ***(9marks)***

10. The following table shows the milk production in litres of a diary farm in each quarter of the years 2012, 2013, and 2014

year	**1st quarter**	**2nd quarter**	**3rd quarter**	**4th quarter**
2012	2490	3640	5015	1480
2013	2360	3520	4890	1375
2014	2155	3265	4630	1170

(a.) Calculate the four point moving averages. ***(6marks)***

(b.) (i.) On the same axes, plot and draw graphs of the original data and the four – point moving averages.

(ii.) comment on the trend of milk production. ***(6marks)***

(c.) Use your graph in (b) to estimate the amount of milk produced by the farm in the 1st quarter of 2015. ***(3marks)***

11. (a.) The probability mass function of s random variable X is given by

$$P(X = x) = \begin{cases} kx + \frac{1}{5} & ,x = -1,0,1,2. \\ 0 \quad , & elsewhere \end{cases}$$

Where k is a constant.

(i.) Determine the value of k

(ii.) Compute the mean

(iii.) Find the mode

(iv.) Find $P(x < 0)$.

(15marks)

12. (a.) Given that $V = 7 + 5t - 3t^2$, find the maximum value of V.

(4marks)

(b.) If $\frac{dy}{dx} = 7$, and $x = 1, y = 10,\ find;$

(i.) y in terms of x

(ii.) x when y = 24 ***(7marks)***

(c.) Evaluate $\int_2^5 \left(\frac{x^2}{3}\right) dx$ ***(4marks)***

13. (a.) Use matrix to solve the simultaneous equations

$$8x - y = 6$$
$$3x + 2y = 26$$

(6marks)

(b.) Mukisa ordered for the following items from a shop; 2 kg of sugar, 3 bars of soap and 1 packet of tea leaves. And Okello ordered for 4 kg of sugar, 1 bar of soap and 2 packets of tea leaves. If the cost of sugar is shs. 2500 per kg, soap shs.2100 a bar and tea leaves shs. 1200 a packet.

(i.) Form a matrix of order 2 X 3 for the items ordered

(ii.) for a matrix of order 3 X 1 for the costs of the items.

(iii.) Use matrix multiplication to find the bills paid by each person.

(9marks)

14. (a.) Two points A and B are 2om apart, if a force of 50N is used to move a body from A to B in 15 s .

(i.) Find the work done.

(ii.) Calculate the rate at which the force is acting. ***(4marks)***

(b.) The point P is 5 metres vertically above point O. A body of mass 0.6 kg is projected from

P vertically downwards with a speed of 3.5ms^{-1}. Find the speed of the body when it reaches O. ***(11marks)***

THE END

SECTION A: (40 MARKS)

Attempt ***all*** questions in this section.

1. Three matrices **P**, **Q** and ***I*** are such that $\boldsymbol{P} = \begin{pmatrix} a & a+1 \\ a-1 & a+2 \end{pmatrix}$ is singular and ***I*** is an identity matrix. Find the value of a and hence the matrix Q if $\boldsymbol{P} + \boldsymbol{I} = \boldsymbol{Q}$. *(05 marks)*
2. Given that A(1,2),B(4,3) and C(5,-1) are vertices of a triangle ABC, find angle ABC. *(05 marks)*
3. If $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $4x^2 - 8x + 1 = 0$, find the equation whose roots are α and β. *(05 marks)*
4. Two bags contain similar balls. Bag A contains 4 red and 3 white balls while bag B contains 3 red and 4 white balls. A bag is selected at random and a ball is drawn from it. Find the probability that a red ball is drawn. *(05 marks)*
5. When a polynomial $g(x)$ is divided by $x^2 + 2x - 3$, the remainder is $2x - 2$. Find the remainder when $g(x)$ is divided by
   (i) $x-1$ *(03 marks)*
   (ii) $x+3$ *(02 marks)*
6. The table below shows the price per kg of three food crops .

Item	Price per kg (sh.)		weights
	2000	2010	
Beans	4000	5000	3
Millet	3000	4000	3
Maize	2500	3000	4

   (a) Calculate the price index of each item for 2010 basing on 2000. *(03 marks)*
   (b) Calculate the weighted price index for 2010. *(02 marks)*
7. The number of computers sold by JA company in a period of 8 months is as shown below.

No. of computers	250	200	220	270	220	260	300	240
month	Jan	Feb	Mar	Apr	May	June	July	Aug

   Calculate the four point moving averages for the data. *(05 marks)*

8. Three forces of magnitudes 5N,12N and 10N on bearings of $060^0, 210^0$ and 330^0 respectively act on a particle . Find the resultant of the system of forces. *(05 marks)*

SECTION B: (60 MARKS)

Attempt only ***four*** questions in this section.

9. The table below shows the cumulative frequency distribution of marks of 800 candidates who sat a national mathematics contest.

Mark(%)	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
F	30	80	180	330	480	610	700	760	790	800

(a) Calculate the mean and standard deviation. *(08 marks)*

(b) Construct an Ogive for the data and use it to estimate the

(i) Median mark *(04 marks)*

(ii) Quartile deviation *(02 marks)*

(iii) Proportion of candidates that failed if the pass mark was 50%. (01 mark)

10. A quadratic curve has gradient function $(k-2x)$ and is such that when $x=1, y=2$ and when $x=-1$, $y=0$.

(a) Find the value of k and state the equation of the curve. *(07 marks)*

(b) Sketch the curve *(05 marks)*

(c) Find the area bounded by the curve and the x - axis. *(03 marks)*

11. The table below gives marks obtained in a mathematics exam (**M**) and physics exam (**P**) obtained by 10 candidates.

(a) (i) Draw a scatter diagram and comment . *(07 marks)*

(ii) Find the score in mathematics by a candidate who scored 82 in physics. *(02 marks)*

(b) Calculate the rank correlation coefficient and comment on your result.

(06 marks)

12. (a) A and B are events such that $P(A)=1/3$, $P(A \text{ or } B \text{ but not both})=5/12$ and $P(B)=1/4$. Calculate

(i) $P(A\cup B)$ *(04 marks)*

(ii) $P(A'\cap B)$ *(02 marks)*

(iii) $P(B'/A)$ *(02 marks)*

(b)Two men fire at a target. The probability that Allan hits the target is $1/2$ and the probability that Bob does not hit the target is $1/3$.Allan fires at the target first followed by Bob. Find the probability that

(i) Both hit the target *(02 marks)*

(ii) Only one hits the target *(03 marks)*

(iii)None of them hits the target. *(02 marks)*

13.(a) Given that $2\sin(A-B)=\sin(A+B;)$

(i) Show that $\tan A = 3\tan B.$ *(03 marks)*

(ii) Hence determine the possible values of A between -180^0 and 180^0 when $B=30^0$. *(03 marks)*

(b) Solve the equation $2\sin 2x - \cos 2x = 1$ for $0^0 \le x \le 360^0$.*(06 marks)*

(c) Without using tables or calculators, show that $\cos 75^0 = \frac{\sqrt{2}(\sqrt{3}-1)}{4}$.

(03 marks)

14.(a) Bodies of mass 6kg and 2kg are connected by a light inextensible string passing over a smooth fixed pulley with the masses hanging vertically. Find the acceleration of the system when released from rest. *(05 marks)*

(b)A body of mass 2kg moves along a smooth horizontal surface with speed of $2ms^{-1}$. It then meets a rough horizontal surface whose co-efficient of friction is 0.2. Find the horizontal distance it travels on the rough surface before it comes to rest. *(05 marks)*

(c) A particle of mass 5kg rests on a smooth surface of a plane inclined at angle of 30^0 to the horizontal. When a force X acting up the plane is applied to the particle, it rests in equilibrium. Find the normal reaction and force X. *(05 marks)*

END

SENIOR 6A & 6B SUBSDIARY MATHS 1(S475/1) REVISION EXERCISES

Students are advised to do as many questions as possible.

SECTION A TYPE OF QUESTIONS. (17 questions)

1. Two events are such that P(A')= $\frac{11}{13}$, P(B')= $\frac{3}{5}$ and P(A $\cap B'$)= $\frac{2}{5}$.

   Find (a) P(A $\cap B$)   (b) P(A $\cup B'$)

2. Differentiate with respect to x; (a) y= x^5 - 4 x^3 - 6   (b) y= - 4 $x^{-2} - \frac{1}{x^4}$

3. Two events A and B are such that P(B)=0.6 and P(A $\cup$B)= 0.94.

   Find (a) P(A) (b) P(A $\cap B'$)

4. Mary takes $1/_{15}$ minutes to cover a distance of 0.03km. If she accelerates at 2m/s^2 ,

   Calculate the; (a) initial speed

   (b) new speed.

5. An Aeroplane lands at Entebbe International airport at 216km/hr. If the plane covers a distance of 1500 m to come to rest, find the;

   (a) time it takes to stop.

   (b) acceleration of the plane.

6. Find the gradient of the curve y= 3x^2 (5x-1) at the point B (2,4).

7. Solve the equation 2Sec$^2\theta$ +3tanθ-3= 0 such that $0° \leq \theta \leq 180°$

8.Solve the equation cosθ = sin2θ for values of θ from 0^0 to 360^0 .

9. The position vectors of points A and B are 3 **i** -5 **j** and 5 **i** +9 **j** respectively. Find the

   (a) position vector of the midpoint M of vector **BA**

   (b) angle between vector **OM** and **OB**

10. Solve the simultaneous equations; **2x -2y =1** and **x^2 - xy - 4=0**.

11. The table below shows the marks(x) and the frequency(f)

	21	23	24	45
f	4	2	3	6

Calculate the (a) mean mark

(b) standard deviation

12. The displacement vector of the particle is given by $\mathbf{r}=4t^2\,\mathbf{i}-5t\,\mathbf{j}$ metres, where t is the time in seconds. Find the speed of the particle after 2 seconds.

13. Use matrices to solve for x and y given that $A=\begin{pmatrix}3 & 2\\ 5 & 4\end{pmatrix}$, $M=\begin{pmatrix}x\\ y\end{pmatrix}$ and $C=\begin{pmatrix}1\\ -4\end{pmatrix}$

such that AM = C.

14. In the crested tower building, a lift is used for regular movements. A Lift is currently parked on the third floor and someone on the seventh floor calls for it, then gets in and travels to the ground floor. Each floor is 3.5m and the whole process takes 50 seconds.

Find the (a) overall average speed of the lift.

(b) overall average velocity of the lift.

15. A car increased its velocity from 5m/s to 72km/hr in a distance of 50m. If the car moved with uniform acceleration, find its;

(a) acceleration

(b) velocity when it had covered 20m.

16. Express $3x^2+4x+6$ in the form $A(x+B)^2+C$, where A, B and C are constants. Hence find the minimum value of the expression.

17. The table below shows the grades scored by a group of seven students.

Maths	E	B	D	C	A	O	D
Physics	O	B	E	D	B	C	A

Calculate the rank correlation coefficient for the data. Comment on your result.

SECTION B TYPE OF QUESTIONS (06 questions)

18.(a) Given that matrix $Q=\begin{pmatrix}1 & -2\\ 3 & 1\end{pmatrix}$, find Q^{-1}. Hence, solve the equations;

x- 2y = - 4 and 3x + y = 9.

(b) Given that α and β are the roots of the equation $\mathbf{2x^2+5x-4=0}$, find the equation whose roots are $\alpha^3\beta$ and $\alpha\beta^3$.

19. The equation of the curve is $\mathbf{y = 3+2x- x^2}$.

(a)Determine the (i) coordinates and nature of the turning point of the curve.

(ii) y and x intercepts of the curve.

(b)(i) Sketch the curve (ii) Find the area enclosed by the curve and the x- axis.

20. The following are the final exam results which were scored by twelve students in Economics (x) and Geography (y)

x	35	56	65	78	49	62	22	90	77	35	52	93
y	57	72	63	76	53	100	38	82	82	19	43	79

(a) Draw a scatter diagram for the data.

(b) Draw the line of best fit and comment on the graph. If x =70, estimate the value of y from the graph.

(c) Calculate the rank correlation coefficient between x and y.

21. The table below shows the ages in years of the mothers at the time they had their first child.

Age	15-	20-	25-	30-	35-	40-45
Frequency	2	14	29	43	33	9

(a) Calculate the mode, mean and the variance of the distribution.

(b) Draw an ogive (cumulative frequency curve) and use it to estimate the

(i) median age, (ii)inter quartile range

22.The table below shows the prices (ug sh) of the items and their corresponding weights.

Item	**Price for year 2000**	**Price for year 2004**	**Weight**
Food	55000	60,000	4
Housing	48,000	52,000	2
Transport	15,000	29,000	3

Using year 2000 as base period, calculate the

(a) price relative for each item

(b) weighted aggregate price index and comment on your result

(c) weighted price index

(d) simple aggregate price index

23. A car travelling on a straight road ABCD starts from rest at A. It travels to B with uniform acceleration until it attains a speed of 12m/s after 2 seconds. It then changes to a uniform acceleration of $1m/s^2$ for 8 seconds until it reaches C. The car then retards to rest at D after a further 10 seconds.

(a) Find the (i) acceleration of the car
(ii) retardation of the car

(b) Sketch the velocity- time graph for the motion and find average speed.

SECTION A (40 MARKS)

Answer all the questions in this section.

1. If $5^x \times 25^{2y} = 1$ and $3^{5x} \times 9^y = \frac{1}{9}$, calculate the value of x and y. (5marks)

2. The table below shows the original marks of six candidates in two examinations.

Candidate	A	B	C	D	E	F
English	38	62	56	42	59	48
History	64	84	84	60	73	69

Calculate the spearman's rank correlation coefficient and comment on the value of your results. (5marks)

3. A and B are acute angles such that $\sin A = \frac{12}{13}$ and $\cos B = \frac{4}{5}$. Without the use of tables or a calculator, find the value of $\sin(A - B)$. (5marks)

4. A committee of 5 people is to be formed from a group of 6 men and 7 women.
   a) Find the number of possible committees. (2marks)
   b) What is the probability that there are only 2 women on the committee? (3marks)

5. The fifth term of an A.P is 23 and the twelfth term is 37. Find the first term and the common difference of the series. (5marks)

6. Three events A, B and C are such that P(A) = 0.6, P(B) = 0.8, P(B/A) = 0.45 and $P(B \cap C) = 0.28$. find
   a) $P(A \cap B)$
   b) $P(C/B)$ (5marks)

7. If α^2 and β^2 are the roots of $x^2 - 21x + 4 = 0$ and α and β are both positive, find;
   i) $\alpha\beta$
   ii) $\alpha + \beta$ (5marks)

8. Find the constant force necessary to accelerate a car of mass 1000 kg from 15 ms^{-1} to 20 ms^{-1} in 10 seconds against a resistance of 270 N. (5marks)

SECTION B (60 MARKS)

Answer only four questions from this section.

9. Eggs laid at Hill farm are weighed and the results grouped as shown below:

Mass (g)	Frequency
– 50	3
– 54	2
– 58	5
– 62	12
– 66	10
– 70	6
– 74	2

a) i) Draw a cumulative frequency curve for the data.

ii) Use the curve to estimate the number of eggs of mass less than 60 g.

b) Calculate the:-

i) median mark

ii) variance (15marks)

10.a) Find the angle between the vectors $\boldsymbol{e} = -i - 2j$ and $\boldsymbol{f} = 2i + j$. Give your answer to the nearest degree.

b) The points A, B and C have position vectors **a**, **b** and **c** respectively referred to an origin O.

i) Given that the point X lies on AB produced so that AB : BX = 2:1, find the position vector of **X** in terms of **a** and **b**.

ii) If Y lies on BC so that BY : YC = 1 : 3, find the position vector of Y, in terms of **b** and **c**. (15marks)

11.a) The masses of packets from a particular machine are normally distributed with mean of 200g and standard deviation of 2g. Find the probability that a randomly selected package from the machine weighs:

i) less than 197g

ii) more than 200.5g

iii) between 198.5g and 199.5g (10marks)

b) Given that $X \sim B[10, 0.8]$, find

i) P(X=8)

ii) the expectation (5marks)

12. a) Sketch the curve $y = x^2 - 6x + 5$, showing all the necessary points. (10marks)

b) Find the area between the curve $y = x^2 - 6x + 5$ and the x-axis from $x = 0$ to $x = 3$. (5marks)

13. A bag contains 5 black counters and 6 red counters. Two counters are drawn, one at a time and not replaced. Let X be the r.v "the number of red counters drawn". Find E(X). (15marks)

14. A motorist moving at 90 kmh^{-1} decelerates uniformly to a velocity V ms^{-1} in 10 second. He maintains this speed for 30 seconds and then decelerates uniformly to rest in 20 seconds.

a) Sketch a velocity – time graph for the motion of the motorist.(6marks)

b) Given that the total distance travelled is 800m, use your graph to calculate the value of V. (5marks)

c) Determine the two decelerations. (4marks)

END

MOUNT OF OLIVES COLLEGE KAKIRI
INTERNAL EXAMINATION 2016
SUBSIDIARY MATHEMATICS

Duration: *2hrs 40mins*
Instructions: *Answer all the eight questions in Section and only four question in Section B*
Any additional questions will not be marked
Begin each answer on a fresh sheet of paper where necessary take acceleration due to gravity as $g = 9.8ms^{-2}$
Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used

SECTION A (40 marks)
Answer all questions in this section

1. Evaluate; $\int_2^3 (6x^2 - 1)dx$ 5mks

2. Solve for x: $\log_3{}^{(2x+1)} - \log_3{}^{(3x-11)} - 2 = 0.$ 5mks

3. If vectors $\boldsymbol{r}_1 = 3\boldsymbol{i} + 2\boldsymbol{j} + 7\boldsymbol{k}$ and $\boldsymbol{r}_2 = 2\boldsymbol{i} + 4\boldsymbol{j} - 5\boldsymbol{k}$.
Find the angle between vectors $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ 5mks

4. The weights of a group of students form a normal distribution with mean 67.6kg and standard deviation 6.2. Find the probability that the weight of a student chosen at random lies between 66kg and 79kg.5mks

5. A random variable X has a probability distribution;

x	0	1	2	3
f(x)	0.1	0.3	0.5	0.2

Calculate the variance of x. 5mks

6. A motor car, starting from rest and moving with uniform acceleration, goes 9.5m in the 10th minute after starting. Find the acceleration of the car, and the distance covered during 5 seconds form the start. 5mks

7. Use matrix method to solve the equations;

$3x + y = 4$

$4y + 26 = 2x$ 5mks

8. The table shows marks obtained in Submaths and Economics for ten students;

Economics	90	40	66	85	50	50	28	75	74	34
Submaths	86	40	64	80	45	56	29	80	67	37

Calculate the rank correlation coefficient for the data. Hence comment on your result. 5mks

SECTION B(60 marks)

Attempt any four questions from this section

9. a) The sum of the 5^{th}, 6^{th} and 7^{th} terms of an arithmetic progression is 95 and 10^{th} term is 49. Find the;

   i) Common difference and first term 7mks

   ii) Sum of the first 22 terms of the progression. 2mks

   b) An employee decided to make monthly savings of his salary by starting with Shs. 60,000 in January 2010. He constantly increased the savings every month by Shs. 5,000. Find the total of his savings at the end of August 2011. 6mks

10. The following table shows the number of pairs of shoes sold by a certain shoe company in each quarter of the years 2013, 2014 and 2015.

Year	Quarter			
	1	2	3	4
2013	234	926	653	431
2014	275	978	704	472
2015	296	1003	728	498

   a) Calculate the four point quarterly moving averages. 5mks

   b) On the same axes, plot and draw graphs of the given data and the four point moving averages. 7mks

   c) Use your graph in (b) to estimate the number of shoes which were sold in the first quarter 2016. 3mks

11. a) Events A and B are independent where $P(A \cap B) = \frac{1}{4}$ and

$P(A \cup B)^1 = \frac{1}{4}$. Find;

i) P(A)

ii) ii) P(B) 7mks

b) At a certain police traffic checking point, the probability that a driver is found drunk is 0.6. Out of 8 drivers checked, find the;

i) Expected number of drunkard drivers

ii) Probability that exactly 3 drivers are found drunk

iii) Probability that more than 6 drivers are found drunk 8mks

12. a) A curve has the equation $y = x^3 - \frac{3}{2}x^2 - 6x + 12$.

i) Write down an expression for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

ii) Find the x – coordinates of the two stationary points on the curve. Hence determine the nature of the stationary values.

10mks

b) Solve the differential equation, $\frac{dy}{dx} = 3 + 9x$, given that y = 15 when x = 2. 5mks

13. The table below shows the age at which women marry in a certain country;

19	20	19	22	28	22
30	31	36	21	29	24
34	33	39	23	26	21
32	18	21	37	25	27
17	35	24	25	27	22
16	38	36	26	38	21

a) Form a frequency distribution table with class intervals of 5 with the lowest age of 16.

b) Calculate;

i) Mean

ii) Modal age

c) Draw a cumulative frequency curve and use it to estimate the percentage of women who marry at the age of 30 and above.

14. a) A block of mass 200kg rests on a rough horizontal plane. The coefficient of friction between the block and the plane is 0.25. Calculate the frictional force experienced by the block when a horizontal force of 50N acts on the block.

b) A force acting on a particle of mass 5kg moves it along a straight line with a velocity of $10ms^{-1}$. The rate at which work done by the force is 50 watts. If the particle starts from rest, determine the time it takes to move a distance of 100m. 8mks

END

SECTION A: (40 MARKS)

*Attempt **all** questions in this section.*

1. Find the value of a if $(x-1)^2$ is a factor of $2x^2-5x^2+4x-1$. Hence solve the equation $2x^3-5x^2+4x-1=0$. *(05 marks)*

2. In a geometric progression, the second term exceeds the first term by 20 and the fourth term exceeds the second by 15. Find the possible values of the first term. *(05 marks)*

3. Given that $\frac{dy}{dx}=\frac{4x-3}{3y^2}$ and that $y(0)=1$, obtain an equation relating x and y. *(05 marks)*

4. Two events A and B are such that $P(A)=P(B), P(A\cap B)=1/3$ and $P(A'\cap B')=1/6$, by use of a venn diagram, find $P(A\cap B')$. *(05 marks)*

5. Show that $b=a^2$ if $\log_a b^2+\log_b a^4=6$ provided $b\neq a$. *(05 marks)*

6. The weights of a large number of children are normally distributed with mean of 16kg and standard deviation of 2 kg.Find the probability of children who weigh between 14 and 16 kg. *(05 marks)*

7. The table below shows the cumulative distribution of marks obtained by 25 students in a test.

marks	0	1	2	3	4	5	6	7	8	9
Frequency	1	2	4	3	a	5	c	2	1	1
Cum. freq	1	3	7	10	13	b	21	23	24	25

Given that the median is 13, find the values of a,b and c. Hence calculate the mean mark. *(05 marks)*

8. Five forces of 1,3,5,6 and 7N act along the lines AB,CB,AD,DB and CD respectively of a triangle ABCD. The direction of the forces is given by the order of the letters. The 6N force makes an angle of 30^0 with DC.Taking AB as horizontal, find the magnitude and direction of the resultant. *(05 marks)*

SECTION B : (60 MARKS)

Attempt only ***four*** *questions from this section. All questions carry equal marks.*

9. (a) The table below shows the probability distribution function of a discrete random variable X.

X	-1	0	1	2
$P(X = x)$	0.4	0.1	0.3	0.2

(i) Calculate $P(X < 2 / X > 0)$ *(04 marks)*

(ii) Obtain the variance of X *(04 marks)*

(b) The pdf of a continuous random variable X is given by

$f(x) = \begin{cases} 3kx^2 ; 1 \le x \le 3 \\ 0 \quad ; \text{elsewhere} \end{cases}$. Find

(i) the value of k *(03 marks)*

(ii) the expectation of X. *(04 marks)*

10. The table below shows the number of years a certain patient has smoked Marijuana and the corresponding percentage scored in an aptitude exam.

Patient	A	B	C	D	E	F	G	H	I	J
No. of smokers	15	22	25	28	31	33	36	39	42	48
Exam score	75	70	72	60	35	57	30	55	50	30

(a) Plot the pairs of values on a scatter diagram and use it to identify the type of correlation between the pairs of values. *(07 marks)*

(b) Calculate the rank correlation co-efficient and advise the public accordingly. *(08 marks)*

11. The table below shows the quarterly revenue in shillings of Paroma limited over a three year period.

Year	1^{st} quarter	2^{nd} quarter	3^{rd} quarter	4^{th} quarter
2008	14,000,000	17,000,000	28,000,000	13,000,000
2009	18,000,000	21,000,000	30,000,000	15,000,000
2010	24,000,000	23,000,000	34,000,000	19,000,000

(a) Plot these data on a graph paper using a suitable axes. *(04 marks)*

(b) Calculate the four quarterly moving averages and plot these on the graph obtained in (a) above. *(07 marks)*

(c) Draw a trend line and use it to estimate the income expected in the first quarter of 2011. *(03 marks)*

(d) Comment on the general trend of the revenue over the three year period. *(01 mark)*

12.(a) Show that $\frac{\cos^2\theta}{1-\sin\theta} = 1+\sin\theta$. Hence solve the equation

$\frac{\cos^2\theta}{1-\sin\theta} = \cos 2\theta$ for $0^0 \le \theta \le 360^0$. *(08 marks)*

(b) Without using tables or calculator, find in surd form the value of $\tan 105^0$. *(04 marks)*

(c) If $\tan\theta = -8/15$ and θ is obtuse, find the value $\cos ec\theta$ without using a calculator. *(03 marks)*

13. A curve y has a gradient function $6-2x$ and it passes through the point $(-1,5)$.

(a) Find the equation of the curve *(05 marks)*

(b) Sketch the curve *(06 marks)*

(c) Obtain the area bounded by the curve and the x-axis. *(04 marks)*

14. A stone Q is projected vertically upwards with a speed of $5ms^{-1}$ from the top of a cliff 10m above the ground. At the same time another stone P is projected vertically upwards from the bottom of the cliff at $12ms^{-1}$. Calculate the

(a) Vertical distance between the two stones when Q is at its maximum. *(07 marks)*

(b) Velocity of P when the first stone is at its maximum. *(02 marks)*

(c) Time that elapses and position jusy before the two stones collide. *(06 marks)*

END